# Vectorial vortex mode transformation for a hollow waveguide using Pancharatnam–Berry phase optical elements

**Yaniv Yirmiyahu, Avi Niv, Gabriel Biener, Vladimir Kleiner, and Erez Hasman**

*Optical Engineering Laboratory, Faculty of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel*

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Transformation and inverse transformation between a free-space linearly polarized beam and the vectorial vortex mode of a circular hollow waveguide by use of Pancharatnam–Berry phase optical elements is proposed. Demonstration was achieved by fabricating GaAs subwavelength gratings and utilizing a 300 $\mu$m diameter hollow metallic waveguide for 10.6 $\mu$m wavelength $CO_2$ laser radiation. The mode transformations and the excitation of a single vectorial mode inside the hollow waveguide were verified by full polarization measurements. In addition, the inverse mode transformation of the single vectorial mode excitation in the waveguide enabled us to experimentally obtain a linearly polarized bright spot with a high central lobe.

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Electromagnetic vectorial modes are characteristic of a specific medium such as hollow waveguides, which are tubes with highly reflective inner surfaces. The hollow waveguide's air core enables broadband as well as high-power transmission. These capabilities make hollow waveguides an important tool in mid-infrared (IR) technology, where optical materials are scarce. For example, hollow waveguides are used for the delivery of $CO_2$ and Er:YAG laser light in industrial and medical applications, as well as in spectroscopic and radiometric measurement. Today, the structure of the hollow waveguide's vectorial modes is well understood.[1,2] For circular hollow waveguides, the low-loss modes are vectorial vortices whose exact structure depends on the reflecting surface's optical properties. Vectorial vortices appear where a scalar vortex is embedded in at least one of the orthogonal polarizations components of a vectorial electromagnetic field.[3] Recently, hollow waveguides have received additional attention due to their potential use in atom guiding,[4] modal dispersion, fiber lasers,[5,6] photonic bandgap fibers,[7] and for their use as self-monitoring devices.[8]

The main drawback of hollow waveguides, however, is that their low-loss modes are optical vortices having dark central cores.[1,2] Therefore, coupling linearly or any other uniformly polarized light to the low-loss modes of a hollow waveguide requires spatial polarization manipulation. Another drawback is the low focusability that results when the low-loss modes of circular hollow waveguides are used. Therefore, the next evolution in hollow waveguide technology will require the ability to transform the free-space scalar modes to the vectorial modes of hollow waveguides, and vice versa. As an outgrowth of this knowledge, we anticipate that the excitation of a single low-loss mode will lead to more efficient hollow waveguide lasers.[6]

Efficient mode conversion requires matching of the phase, amplitude, and polarization state. Phase- and amplitude-matching methods are well developed and can be achieved by conventional optical devices, irregular waveguides, diffractive optics, or holographic methods. The main problem remains matching the polarization states, which requires polarization manipulation techniques. Several techniques exist, such as liquid crystal spatial light modulators, interferometric techniques, and the use of lasers with intracavity phase elements.[9] However, all these methods are either cumbersome, have low power thresholds, or are inadequate in the IR regime.

Recently, we demonstrated spatial polarization manipulation by space-variant subwavelength gratings. The optical properties of these devices stem from the geometric Pancharatnam–Berry phase. Thus, they are called Pancharatnam–Berry phase optical elements (PBOEs).[10] PBOEs are compact optical devices with high power durability. They were used for the formation and investigation of propagation-invariant vectorial Bessel beams,[10] rotating vectorial vortices,[11] and for the excitation of a vectorial hollow waveguide mode in the 1.55 $\mu$m wavelength regime.[12]

In this Letter, we present the transformation of a free-space linearly polarized beam to the hollow waveguide's azimuthally polarized vectorial mode and its inverse transformation by use of PBOEs. Experimental demonstration is obtained by coupling a linearly polarized 10.6 $\mu$m wavelength $CO_2$ laser radiation to a single vectorial mode of a 300 $\mu$m diameter hollow metallic waveguide. The transformation and excitation of a single vectorial mode was confirmed by full polarization measurement. We also demonstrated that single-mode operation and inverse mode transformation produce a lineraly polarized bright spot with a high central lobe.

The concept of our method is illustrated in Fig. 1. The linearly polarized plane wave emerging from the laser impinges on a PBOE (PBOE1) that transforms its linear polarization orientation to an azimuthally polarized vectorial vortex. This operation serves to

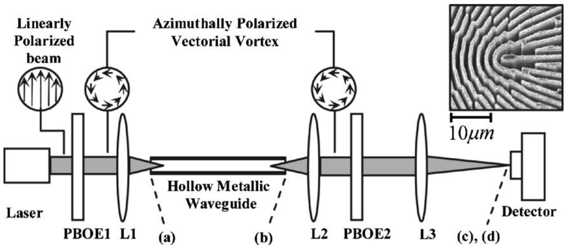

Fig. 1. Concept of vectorial vortex mode transformation and inverse transformation for a hollow metallic waveguide mode using PBOEs. Inset, scanning electron microscope image of the PBOE. Abbreviations defined in text.

match the uniformly polarized free-space beam to the vectorial hollow fiber mode. The vectorial vortex is then coupled to the waveguide by a suitable focusing lens (L1). The light emerging from the waveguide is focused by use of a 4-$f$ system comprising of two lenses (L2 and L3). A second PBOE (PBOE2) is inserted into the Fourier plane of the telescope to inversely transform the azimuthally polarized vectorial vortex to a linearly polarized beam, thereby producing a bright spot.

The general modes of a circular hollow waveguide are the hybrid $\mathrm{HE}_{nm}$, $\mathrm{EH}_{nm}$ and transverse $\mathrm{TE}_{0m}$, $\mathrm{TM}_{0m}$, where the integers $n$, $m>0$ denote the azimuthal and radial mode orders, respectively.[2] The $\mathrm{TE}_{0m}$ modes are azimuthal linearly polarized vectorial vortices with dark central cores.[2] For small values of $m$, these modes possess the least amount of loss and are stable under perturbations in the waveguide structure or variances in the material's optical properties. For this reason, they are the most reasonable choice for single-mode applications.

The Jones representation of the $\mathrm{TE}_{0m}$ modes is given by[1]

$$|E_{0m}\rangle = J_1(k_m r)\exp(i\gamma z)|\varphi\rangle, \tag{1}$$

where $J_1$ is a first-order Bessel function of the first kind, $|\varphi\rangle = i[\exp(i\varphi)|L\rangle + \exp(-i\varphi)|R\rangle]/\sqrt{2}$ denotes an azimuthally polarized vectorial vortex, $|R\rangle = (1, -i)^T/\sqrt{2}$ and $|L\rangle = (1, i)^T/\sqrt{2}$ are the helical basis unit vectors, $(r, \varphi, z)$ are the cylindrical coordinates, and the superscript $T$ denotes a vector transposition. The propagation constant, $\gamma$, is determined by solving the equation that is characteristic of the specific hollow waveguide being used.[1] Finally, the transverse spatial frequency $k_m^2 = k^2 - \gamma^2$ ($k$ is the wavenumber) determines the number of annular Bessel intensity fringes inside the hollow waveguide as well as the electric field amplitude on the waveguide's inside surface.

The polarization manipulations involved in the mode transformation are performed by using PBOEs. A PBOE is considered a constant retardation wave plate for which the fast axis is changed along the face of the device. It is convenient to form such space-varying wave plates by using subwavelength gratings with constant periods and spatially varying groove orientations. Assuming a local retardation of $\pi$ rad and linearly polarized illumination, the Jones vector of the beam that emerges from a PBOE is given by[10]

$$|E_{\mathrm{PBOE}}\rangle = \mathrm{circ}(r/r_0)\frac{i}{\sqrt{2}}[\exp(i2\theta)|L\rangle + \exp(-i2\theta)|R\rangle], \tag{2}$$

where $r_0$ is the radius of the PBOE aperture and $\theta(r, \varphi)$ is the local subwavelength groove orientation. The $|E_{\mathrm{PBOE}}\rangle$ state comprises two scalar waves with orthogonal circular polarizations. The phase factors of the $|R\rangle$ and $|L\rangle$ components result from the space-variant polarization-state manipulation induced by the PBOE and thus are geometrical in nature.[10] As a result, the phase factors depend on the local subwavelength groove orientation alone. The absence of wavelength dependence in the geometrical phase suggests the possibility of using PBOEs as achromatic devices. Setting the local subwavelength groove orientation at $\theta(\varphi) = \varphi/2$ produces the desired vectorial vortex $|\varphi\rangle$.

While the main concern of this Letter is the polarization-state mode transformation, we also needed to direct our attention to matching the phase and amplitude, which we achieved to a satisfactory degree by simply focusing the vectorial vortex. The field at the focus is the Fourier transform of Eq. (2) with $\theta(\varphi) = \varphi/2$, hence $|E_{\mathrm{in}}(r, \varphi)\rangle = [k\exp(ikr^2/2f)]/(i2\pi f r^2)\int_0^{\alpha r} x J_1(x)\mathrm{d}x|\varphi\rangle$, where $\alpha = kr_0/f$ and $f$ is the lens focal length. Analysis of the radial dependence of $|E_{\mathrm{in}}(r, \varphi)\rangle$, as provided by the integral, shows concentric rings with a dark central spot. Also, $|E_{\mathrm{in}}(r, \varphi)\rangle$ maintains the required vectorial vortex polarization structure. Therefore, $|E_{\mathrm{in}}(r, \varphi)\rangle$ is close to $|E_{0m}(r, \varphi)\rangle$, the hollow waveguide mode given by Eq. (1). To avoid edge losses due to imperfections at the entrance of the hollow waveguide, the focused beam was chosen to match the second low-order mode, $\mathrm{TE}_{02}$. Consequently, the light intensity was distributed closer to the center, and edge losses were reduced.

If the mode transformation is represented by a Jones matrix $\mathbf{T}$, then the inverse mode transformation is represented by $\mathbf{T}^{-1}$.[10] In our case, $\mathbf{T} = \mathbf{T}^{-1}$, and inverse mode transformation can therefore be obtained by simply using an identical PBOE after a collimating lens.

To evaluate our method experimentally, we realized PBOEs with a discrete function of the desired spatially varying groove orientation, a 2 $\mu$m subwavelength grating period with a 5 $\mu$m groove depth and 1 cm diameter upon GaAs wafers for use with 10.6 $\mu$m wavelength $\mathrm{CO}_2$ laser radiation. The inset in Fig. 1 shows a scanning electron microscope image of these PBOEs. The space-variant groove orientation is clearly observed. Additional details regarding design considerations and the fabrication process can be found in Ref. 10. The circular hollow waveguide used for this demonstration was a 10 cm length silver/silver iodide (Ag/AgI) hollow silica waveguide with a bore diameter of 300 $\mu$m (Polymicro HWCA300750). Measurements were taken using a pyroelectric IR

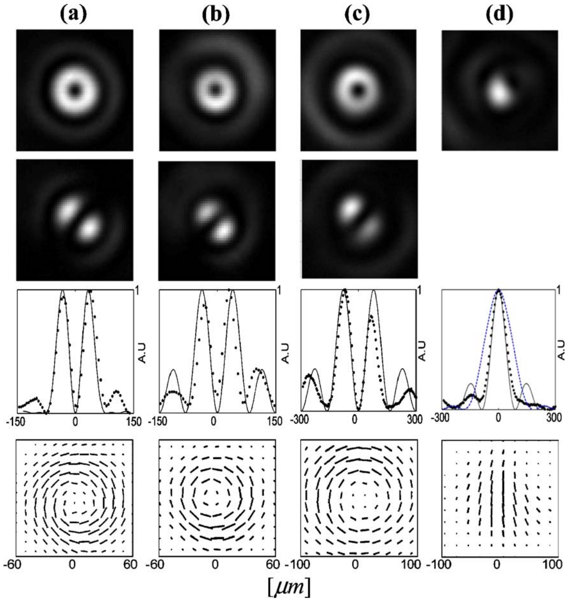

Fig. 2. (Color online) (a) and (b) Fields at the entrance and exit of the hollow metallic waveguide, respectively, for mode value $m=2$, (c) after a 4-$f$ system without a PBOE, (d) with the PBOE in the Fourier plane. The first, second, and third rows show the intensity, intensity after a polarizer, and the measured (dots) and calculated (solid curve) intensity cross sections, respectively. The cross section of Fig. 2(d) contains the calculated result for mode value $m=1$ (blue dashed curve). The fourth row shows the measured polarization orientation of the beam. The respective locations along the setup are also indicated in Fig. 1.

camera (Spiricon Pyrocam III), and polarization data were obtained using the four measurement method.

Figure 2 shows the experimental results. The first and second rows show the captured intensity without and with a polarizer, respectively. The third row shows measured (dots) and calculated (solid curve) intensity cross sections. Finally, the fourth row shows the measured polarization orientation. Figures 2(a) show the field at the waveguide's entrance, obtained by using a lens of 1″ focal length (L1). Two annular intensity rings are clearly observed. The intensity behind a polarizer indicates the vectorial vortex structure.[3] The measured polarization orientation at the waveguide's entrance clearly shows the existence of a vectorial vortex with the desired azimuthal polarization. The good agreement between the measured and calculated cross sections indicates that the desired intensity distribution was indeed obtained. Figures 2(b) show the field at the waveguide's exit. Bessel intensity rings can be distinguished, and an azimuthally polarized vectorial vortex is revealed, indicating the excitation of a single low-order mode inside the hollow waveguide. Note that the theoretical calculation of the cross section was performed by use of Eq. (1) with the mode value $m=2$.

To demonstrate the inverse transformation of the vectorial vortex mode to obtain a linearly polarized bright spot, we used a 4-$f$ system consisting of a 1″ focal length collimating lens (L2) and a focusing lens of 2.5″ focal length (L3). Figures 2(c) show the focus in the case where the second inverse transformer–PBOE was removed. Due to the existence of a vectorial vortex, the beam suffers low focusability, as indicated by the dark central spot. Figures 2(d) show the focus when the second PBOE was reinserted in the Fourier plane of the 4-$f$ system. The observed linearly polarized bright spot with a high central lobe indicates the improved beam focusability resulting from the inverse transformation process. A typical measured deviation of the polarization state's ellipticity was found to be 0.174 rad. Note that the sidelobes in the spot are due to the Bessel distribution of the fiber mode, whereas the magnitude of the sidelobes is dependent on the mode value $m$ given in Eq. (1). Low ring-shaped sidelobes that contain only a small portion of the total power were calculated for a mode value $m=1$ and are shown in the cross section of Fig. 2(d). By using phase and amplitude beam shaping, it is possible to decrease the sidelobes.[13]

We have experimentally demonstrated the transformation and inverse transformation of linearly polarized light at a wavelength of 10.6 $\mu$m to a vectorial mode within a metallic-coated hollow waveguide by use of a PBOE. In the same manner, it is possible to excite and manipulate other low- and high-order vectorial modes of various types of hollow waveguide. The design of PBOEs for this purpose is currently being studied. We also aim to utilize the achromatic properties of PBOEs to produce polychromatic and short-pulse transmission in hollow waveguides.

E. Hasman's e-mail address is mehasman@tx.technion.ac.il.

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