Curved holographic elements for optical coordinate transformations

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A new method for performing optical coordinate transformation is presented. It is based on a curved holographic element on which the interference pattern of two perpendicular plane waves is recorded. The design procedures and results for two curved elements that perform a one-dimensional logarithmic coordinate transformation and a Gaussian-to-uniform-beam conversion are given.

Space-variant coordinate transformations (CT's) on two-dimensional pictures have many applications in optical data processing.[1] One approach for obtaining such CT's exploits reflective[2] or refracting[3] aspheric surfaces. The main difficulty in this approach is its high sensitivity to the accuracy of the aspheric surfaces. Specifically, small deviations of the local slope of the surface may cause severe aberrations in the transformation.

A more common approach for obtaining optical CT's involves planar holographic elements with complicated aspheric phase functions.[4,5] As a result, the holographic elements must be recorded as computer-generated holograms. Unfortunately, with the possible exception of e-beam plotters, all plotters that are used to record computer-generated holograms have either limited resolution or a low space–bandwidth product. These limitations of the plotters directly affect the compactness and the space–bandwidth product capabilities of the optical system that performs the CT.[6]

In this Letter we present a different approach for obtaining CT's, which involves curved holographic elements. Here the curvature provides an additional degree of freedom that enables us to exploit holographic elements with simple phase functions that can be recorded directly by optical means. The design method for such curved holographic elements is described, and it is then illustrated for two specific cases: a logarithmic coordinate transformation and a conversion of a Gaussian beam into a uniform one.

Consider a coordinate transformation for a one-dimensional function $f(x)$,

$$f(x) \to f[u(x)], \qquad (1)$$

where $u(x)$ is the new coordinate. Such a transformation is illustrated graphically with the aid of Fig. 1, where the input function $f(x)$ is drawn along the x axis and the output function $f[u(x)]$ is drawn along the z axis (for simplicity we use a binary function). From each point x_i on the x axis, a vertical line is drawn, and from its intersection with the curve $z = u(x)$ a horizontal line is drawn that intersects the z axis at the point $z_i = u(x_i)$.

The transformation of Fig. 1 may be realized directly by optical means as shown schematically in Fig. 2. A transparency with transmittance function $f(x)$ at the input plane is illuminated by a coherent plane wave. If we assume no diffraction of this plane wave by the input pattern (geometrical shadow approximation[7]), the wave front after the transparency can still be described by parallel rays, where the intensity of each ray is $f(x)$ and x is the coordinate of the ray. These parallel rays are diffracted by a holographic element along the curve $z = u(x)$ by exactly 90° and hence arrive at the output plane at the desired location $z = u(x)$. The curved holographic element may be generated optically by simply recording the interference pattern of two perpendicular plane waves on a curved recording film. Note that the geometrical shadow approximation is valid when the distance between the input and the output planes is small compared with $\lambda^{-1}f_{\max}^{-2}$, where $f_{\max}$ is the maximal spatial frequency of the input.[6]

The curved hologram approach may be generalized for data with two spatial dimensions. Quasi-one-dimensional CT's of the form $[x, y] \to [u(x, y), y]$ can be achieved with the same configuration as for the one-dimensional case, except that now the curved holographic element is located on a surface $z = u(x, y)$. A general two-dimensional CT of the form $[x, y] \to [u(x, y), v(x, y)]$, on the other hand, requires two cascaded curved holograms. The first generates the CT $[x, y] \to [u(x, y), y]$ as discussed above, while the second generates the CT $[u(x, y), y] \to [u(x, y), w(u, y)]$, where $w(u, y) = v(x, y)$.

A sufficient and necessary condition for the validity of the curved hologram approach is that the CT must be monotonic, as can be seen directly from Fig. 2. For the one-dimensional case this condition can be expressed as $du/dx \neq 0$, and for the two-dimensional case it can be expressed as $J \neq 0$, where J is the Jacobian of the transformation.

To evaluate the performance of the curved hologram approach, we performed an experiment for a one-dimensional logarithmic transformation $[x, y] \to [\ln(x), y]$. The optical arrangement for recording the holographic element is shown in

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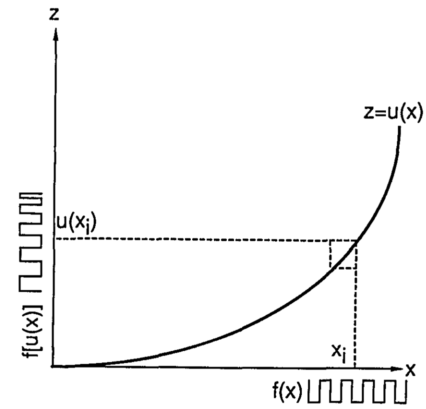

Fig. 1. Graphical illustration of a one-dimensional CT $f(x) \to f[u(x)]$.

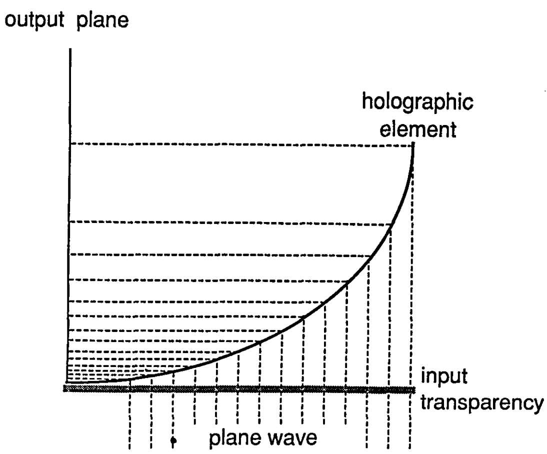

Fig. 2. Optical configuration for fabricating a one-dimensional CT using a curved holographic element.

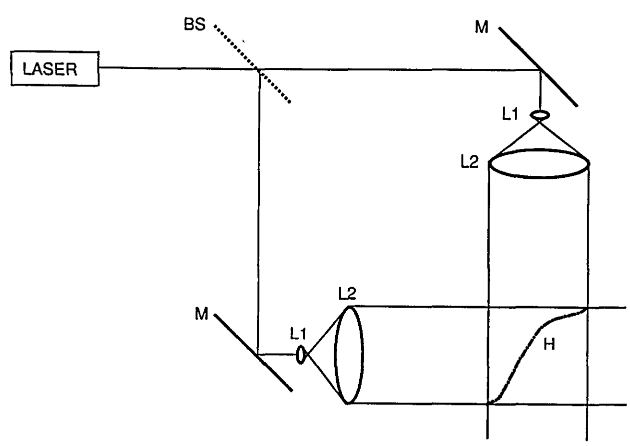

Fig. 3. Optical arrangement for recording the curved holographic element. BS, beam splitter; M's, mirrors; L1 and L2, lenses forming a telescope; H, curved holographic film.

Fig. 3. A laser beam derived from an argon laser is divided by a beam splitter to obtain two beams that are reflected and expanded to form two perpendicular plane waves. The holographic element was recorded on Agfa 8E56 film that was attached to a logarithmic-shaped surface and located in the overlap volume between the two plane waves; the logarithmic sloping was formed by a computer-controlled lathe.

For readout, we exploited the same arrangement of Fig. 3 except that one arm was blocked. The developed film was returned to its original location on the logarithmic surface, and a transparency, containing a 50 mm × 80 mm input scene composed from black and transparent lines whose width formed a geometric series, was inserted in the unblocked arm. The diffracted light along the path of the blocked arm formed the transformed output. The results of the logarithmic CT are shown in Fig. 4. The input is shown in Fig. 4(a), and the corresponding transformed output is shown in Fig. 4(b). As expected from a logarithmic transformation, the output in this case is composed of lines that have the

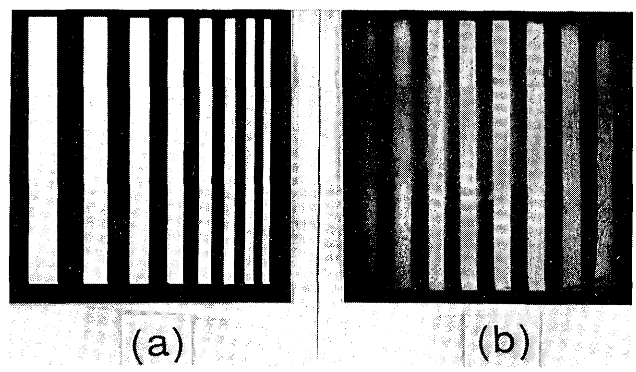

Fig. 4. Experimental results of the one-dimensional logarithmic CT: (a) input, (b) output.

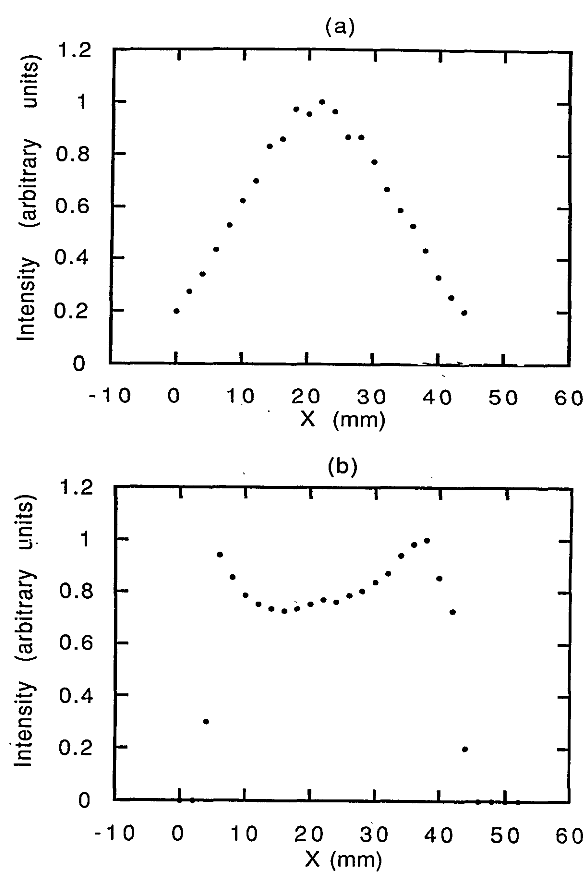

Fig. 5. Experimental intensity cross section for the input and output of the Gaussian-to-uniform-beam conversion: (a) input, (b) output.

same width. The measured distortions of the output were relatively low, less than 0.2 mm, and are probably due to mechanical distortion of the recording film.

We also tested the curved hologram approach to transform a Gaussian beam into one that is uniform in one of the dimensions. Specifically, the input Gaussian beam had an intensity cross section of $I_{in}(x, y) = I_0 \exp[(x^2 + y^2)/r_0^2]$, whereas the output beam was uniform in the x direction, i.e., $I_{out}(x, y) \propto \exp(y^2/r_0^2)$. Such a conversion may be obtained by the following CT (Ref. 8):

$$[x, y] \rightarrow [\mathrm{erf}(\sqrt{2}x/r_0), y], \qquad (2)$$

where erf is the error function (the integral of the Gaussian function). The same procedure was used for recording the curved holographic element as for the logarithmic transformation described above, except that now the shape of the recording surface was that of an error function. The transformation results for an input Gaussian beam with $r_0 = 17$ mm are shown in Fig. 5. The input with its Gaussian shape is shown in Fig. 5(a), whereas the transformed output is depicted in Fig. 5(b). The intensities of the input and output beams were measured by a scanning-knife method,[9] so the y dependence was automatically integrated. As can be seen, the output intensity is indeed uniform in the x direction, up to ~20% accuracy. The deviations may be attributed to mechanical distortion of the recording film or to changes in the diffraction efficiency of the holographic grating at different areas on the film.

In conclusion, a new method for performing optical coordinate transformation on two-dimensional data was presented. It is based on a curved holographic element that is recorded optically in a relatively simple way. Therefore, computer-controlled plotters with extremely high resolution are not needed. Furthermore the curved holographic element diffracts the light by an angle of 90°, a much larger diffraction angle than most planar computer-generated holograms are capable of. Thus, for a given input size, the distance between the input and output planes will be much smaller for the curved hologram than for the planar one. Besides making the total optical system more compact, this shorter distance improves considerably the optical performance of the CT. Specifically, the space–bandwidth product capabilities of the transformation are inversely proportional to the square root of this distance.[6]

The curved holographic element has another advantage over the planar one in those cases where the phase of the output wave front is required to be uniform (a collimated beam). In the planar approach another element is required to collimate the beam, whereas in the curved element approach the beam is inherently collimated in the output plane.

To illustrate the validity of our approach, two curved holographic elements for performing a one-dimensional logarithmic CT and a Gaussian-to-uniform-beam conversion were designed, fabricated, and tested experimentally. In both cases the elements were curved in one dimension only, so it was possible to attach a conventional holographic film to the curved surfaces. For more general transformations, where the elements are curved in two dimensions, it will be necessary to coat the curved surfaces directly with recording materials; for example, by dip-coating technology.

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