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Efficient formation of nondiffracting beams with uniform intensity along the propagation direction

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A technique for forming nondiffracting beams having essentially constant intensity along the propagation direction is presented. In this technique two phase-only holographic elements are exploited to form the nondiffractive beam as well as to obtain a high diffraction efficiency. The elements were designed and formed as computer generated holograms, and the combination was tested experimentally, showing a good agreement with the predicted behavior.

1. Introduction

A new class of solutions to the scalar wave equation, introduced by Durnin [1], reveals the existence of nondiffracting beams, having highly localized intensity distributions that do not change along the propagation distance z, so $I(x, y, z)=I(x, y, z=0)$. Although these solutions are not square integrable, and therefore may not be realized in a rigorous sense, finite aperture approximations exhibit the main propagation features of true nondiffracting beams over the long distances [1,2]. The simplest nondiffracting beam is a monochromatic wave propagating in the z direction with a field amplitude

$$u(r, z; k)=\exp(\mathrm{i}\beta z)\, J_0(\alpha r)\,, \tag{1}$$

where $r=\sqrt{x^2+y^2}$ is the radial coordinate, $k=\sqrt{\alpha^2+\beta^2}$ is the wave number and J_0 is the zero order Bessel function of the first kind. An experimental arrangement for the formation of such a Bessel beam involves a circular slit located in the focal plane of a lens [1]. Unfortunately, in this experimental arrangement, only a slight fraction of the incoming energy is used to generate the desired Bessel beam, resulting in an extremely low light efficiency. To overcome this problem, the use of an axicon for formation of nondiffracting beams was suggested [3]. But these beams have intensity distributions that grow linearly as a function of z, and, consequently, are not strictly nondiffracting.

In this paper we present an approach in which two holographic elements (HEs) are combined to efficiently form nondiffracting beams with a constant intensity along the propagation direction z. One element transforms the uniform intensity distribution of an incident beam to a $1/r$ intensity distribution, whereas the other element combines the correction for the phase distortions that are introduced by the first element, and the desired conical phase transformation of an axicon.

2. Basic relations

The axicon for forming nondiffractive beams can be a holographic element having a conical phase transmission function, given as

$$t(r)=\exp(\mathrm{i}2\pi r/r_0)\,, \qquad r\leqslant R\,,$$
$$\quad =0\,, \qquad r>R\,, \tag{2}$$

where R is the radius of the element and r_0 is a constant. When such an element illuminated with a uniform plane wave, the output light intensity can be calculated by the Fresnel diffraction integral in cylindrical coordinates, as

$$I(r,z)=\left(\frac{2\pi}{\lambda z}\right)^2\left|\int_0^R t(r')\exp[i2\pi(r'^2/2\lambda z)]\right.$$

$$\left.\times J_0(2\pi r r'/\lambda z)r'\,dr'\right|^2, \quad (3)$$

where $t(r')$ is given in eq. (2), and λ is the wavelength of the light. Eq. (3) may be solved analytically by exploiting the stationary phase approximation, to yield [4]

$$I(r,z)\propto z|J_0(2\pi r/r_0)|^2 . \quad (4)$$

Eq. (4) approximates the intensity of a nondiffracting solution, except for a linear growth as a function of z.

To check the validity of the stationary phase approximation, the light intensity was also calculated by solving the Fresnel integral of eq. (3) numerically. Fig. 1 shows a typical solution for the light intensity along the optical axis as a function of the propagation distance z, for $R=5$ mm, $r_0=0.5$ mm and $\lambda=514.5$ nm. It can be seen that beside some local fluctuations, the general trend is indeed of a linear growth of the intensity as a function of z; the fluctuations may be attributed to the diffraction from the aperture ("Fresnel rings"). This linear growth may be explained in terms of geometrical optics. The neighborhood of a point at a distance z along the optical axis, receives light mainly from a thin annulus in the axicon plane, as shown in fig. 2. The radius and area of the annulus, and thereby its energy contribution, are all proportional to z. In order to equalize the energy contribution of all the annuli, so as to remove the linear growth of the axial light intensity, we purpose to appodize the intensity by a factor $1/r$ ($1/\sqrt{r}$ in light amplitude).

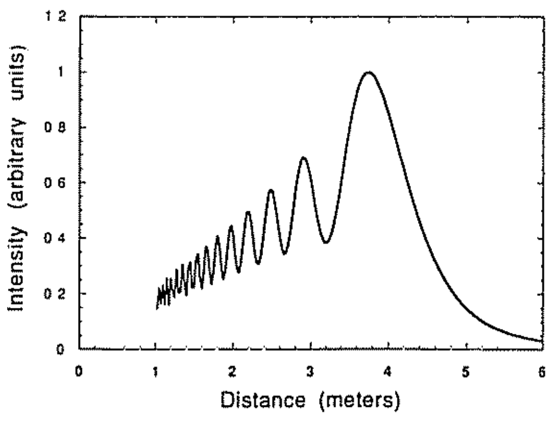

Fig. 1. The calculated light intensity along the optical axis as a function of the distance from an axicon.

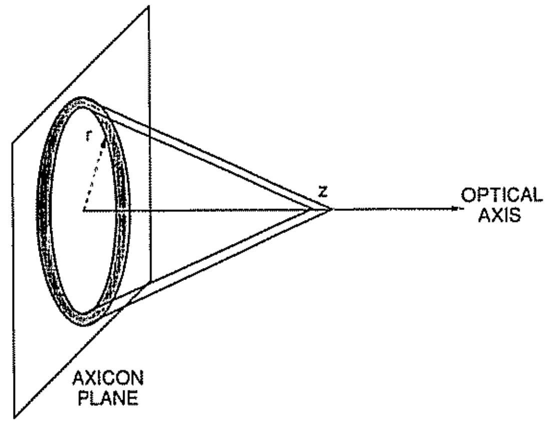

Fig. 2. Geometrical optics description of a typical annulus in the axicon. r, the radius of the annulus; z, the distance where the rays from the annulus intersect the optical axis.

In order to evaluate the suggested appodization, let us consider a unit-amplitude plane wave illuminating a circular HE of radius R that is characterized by the complex amplitude transmission function, as

$$t(r)=(1/\sqrt{r})\exp(i2\pi r/r_0), \quad r\le R,$$
$$=0, \quad r>R. \quad (5)$$

This transmission function has the same conical phase as that of the axicon in eq. (2), but an additional amplitude term of $1/\sqrt{r}$. Again, the light intensity behind the HE can be calculated analytically by following the same procedure that was used to solve eqs. (2) and (3). This yields

$$I(r,z)\propto|J_0(2\pi r/r_0)|^2 . \quad (6)$$

From eq. (6) it is evident that the intensity no longer grows as a function of z; hence a true nondiffracting beam is indeed produced.

To verify the analytic solution of eq. (6), we also solved the Fresnel diffraction integral of eq. (3) with the new transmission function $t(r)$ of eq. (5) numerically; again, the parameters were $R=5$ mm, $r_0=0.5$ mm and $\lambda=514.5$ mm. The calculated results of the light intensity along the optical axis as a function of z are shown in fig. 3. It is evident that aside from the Fresnel fluctuation that are still present, the intensity as a function of z is now essentially constant.

3. Improving the light efficiency

An element having the complex-amplitude transmission function of eq. (5) suffers from a lower light efficiency than the axicon. This reduction in efficiency is caused by the appodization factor $1/\sqrt{r}$. In order to improve the light efficiency, it is possible to replace the single HE having the transmission function of eq. (5) with two phase-only elements. With such a combination, a diffraction efficiency close to 100% can be obtained, either in thick phase material, such as dichromated gelatin [5], or as surface relief kinoforms [6].

The optical arrangement for the two elements is shown in fig. 4. The first HE generates the $1/r$ intensity distribution by performing the proper coordinate transformation (CT) on the incoming plane wave, while the second HE compensates for the phase distortion introduced by the first HE, and adds a conical phase $\exp(-i2\pi r/r_0)$.

An appropriate CT for the first HE may be defined by the 1D transformation, as

$$r \to \rho(r) , \tag{7}$$

where we assumed cylindrial symmetry, and r and ρ

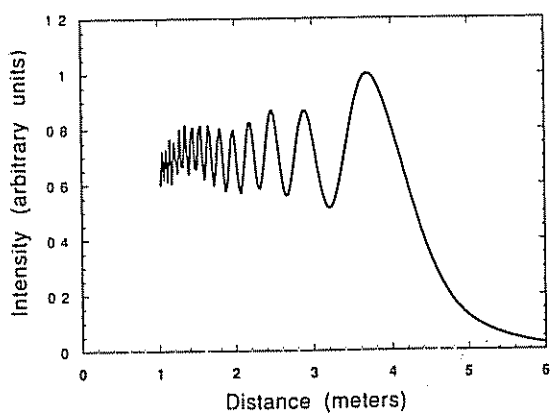

Fig. 3. The calculated light intensity along the optical axis as a function of the distance from an axicon with $1/\sqrt{r}$ appodization.

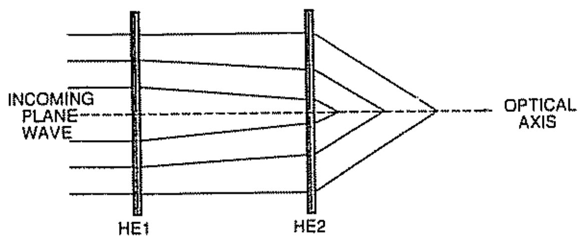

Fig. 4. Optical arrangement for efficient formation of nondiffracting beams. HE1 and HE2 are two phase-only holographic elements. Only some of the light rays are shown.

are the radial coordinates in the planes of the first and second HEs, respectively. To determine the desired $\rho(r)$ for transforming a wavefront with intensity $I_1(r)$ to a wavefront with intensity $I_2(\rho)$, we use the conservation of energy relation in cylindrical coordinates, to yield

$$I_1(r) r \, dr = I_2(\rho) \rho \, d\rho . \tag{8}$$

Assuming an input wavefront with uniform intensity, $I_1(r) = \text{constant}$, a desired output wavefront with an intensity distribution of $I_2(\rho) = 1/\rho$ and performing the necessary integration, eq. (8) yields

$$\rho(r) = r^2/R , \tag{9}$$

where we assumed that the CT maintains the size of the original beam, so the boundary conditions of the integration were $\rho(0) = 0$ and $\rho(R) = R$.

The grating vector (normalized) for a HE that realizes such a CT, within the paraxial approximation, is [7]

$$K_{h1}(r) = \frac{\rho(r) - r}{z_0} = \frac{r^2/R - r}{z_0} , \tag{10}$$

where z_0 is the distance between the two HEs, and an incoming plane wave was assumed. The phase function for this HE is then obtained by integrating the grating vector [7], to yield

$$\phi_{h1}(r) = \frac{2\pi}{\lambda} \int_0^r K_{h1}(r') \, dr' = \frac{2\pi}{\lambda} \frac{r^3/3R - r^2/2}{z_0} , \tag{11}$$

where the integration constant was chosen as zero.

Now, the phase function of the second HE is comprised of two terms. The first compensates for the phase distortion caused by the first HE, while the second is simply a conical phase. To determine the first term we again resort to the grating vector description, in which the phase compensation requirement is expressed as

$$K_{h2}(\rho) = -K_{h1}[r(\rho)] , \tag{12}$$

where K_{h2} is the grating vector of the second HE. Inserting eqs. (9) and (10) into eq. (12), yields

$$K_{h2}(\rho) = (\rho - \sqrt{R\rho})/z_0 . \tag{13}$$

Integrating this grating vector and adding to it the conical phase, yields the complete phase function of the second HE, as

$$\phi_{h2}(\rho) = \frac{2\pi}{\lambda} \frac{\rho^2/2 - \frac{2}{3}\rho^{3/2}\sqrt{R}}{z_0} + \frac{2\pi\rho}{\rho_0} , \tag{14}$$

where ρ_0 is a constant identical to r_0 of eq. (2).

4. Experimental procedure and results

In order to verify our theoretical predictions, we performed an experiment using the arrangement depicted in fig. 4. The two HEs, having the phase function of eq. (11) and eq. (14), respectively, were realized as Lee-type binary computer generated holograms with diffraction efficiency of 10.5% each [8], higher diffraction efficiencies, up to 100% are possible with surface relief kinoforms [6]. The distance between the two HEs was chosen as $z_0 = 2000$ mm, and as before $R = 5$ mm and $r_0 = 0.5$ mm. Finally, a linear phase term of $2\pi x \sin\theta_x/\lambda$ was added to the phase functions of eq. (11) and eq. (14); the off-axis angle θ_x was chosen as 1/90 radians where x is one of the transverse coordinates.

For the experiment, the first HE was illuminated with a plane wave derived from an argon laser at $\lambda = 514.5$ nm, and the beam profile was measured at several distances from the second HE with a CCD camera. The results are presented in figs. 5 and 6. Figs. 5 a,b show the beam profile at a distance of 1.5 m and 4 m, respectively, from the second HE. As shown, besides some electronic noise that is present, the two beam profiles are almost identical. Furthermore, the width of the central lobe is approximately 0.4 mm, and the ratio of the central lobe to the first sidelobe is approximately 6:1, in good agreement to the theoretical prediction of eq. (6). Fig. 6 present the measured axial intensity of the beam up to a distance of 5 m. As can be seen, the axial intensity changes by less than 20%, for a distance ranging from 1.5 to 4 m; for such a range of distances the intensity grows by approximately a factor of three for an axicon element. The 20% changes are due, in part, to the predicted fluctuations that are caused by the diffraction from the aperture (Fresnel rings), and, in part, to misalignment of the optical components, nonuniformities in the input beam intensity, and thickness variations of the photographic plates. In any case, the experimental results are in good agreement with the predicted behavior.

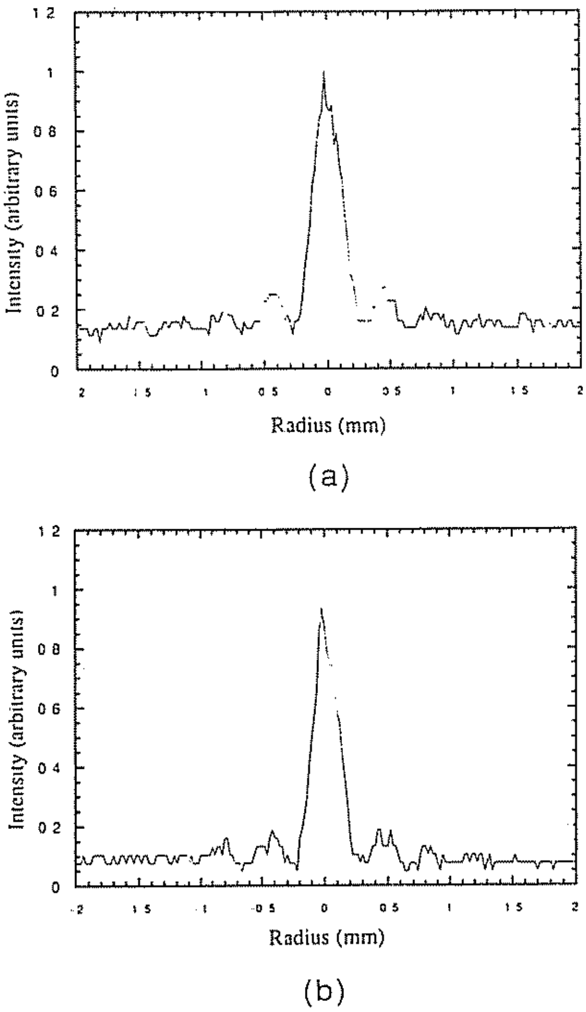

Fig. 5. Experimental beam profiles at two distances from the second holographic element: (a) 1.5 m, (b) 4 m.

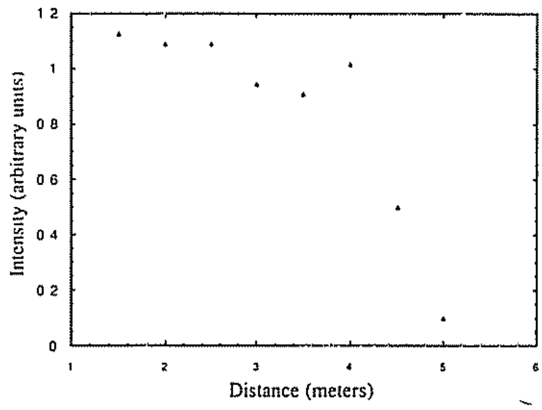

Fig. 6. Experimental light intensity along the optical axis as a function of the distance from the second holographic element.

5. Concluding remarks

We presented an approach for optically implementing a truly nondiffracting beam, that has essentially a constant intensity along the optical axis. This approach involved two phase-only holographic elements, so high diffraction efficiencies are possible. Finally, the approach can be readily extended to obtain any desired axial intensity distribution of the nondiffractive beams, by properly choosing the apodization of the intensity that is derived from the first holographic element.

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