Polarization dependent focusing lens by use of quantized Pancharatnam–Berry phase diffractive optics

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Quantized Pancharatnam–Berry phase diffractive optics using computer-generated space-variant subwavelength dielectric grating is presented. The formation of the geometrical phase is done by discrete orientation of the local subwavelength grating. We discuss a theoretical analysis and experimentally demonstrate a quantized geometrical blazed phase of polarization diffraction grating, as well as polarization dependent focusing lens for infrared radiation at wavelength 10.6 μm. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1539300]

One of the most successful and viable outgrowths of holography involves diffractive optical elements (DOEs). The DOEs diffract light from a generalized grating structure having nonuniform groove spacing. They can be formed as thin optical elements that provide unique functions and configurations. High diffraction efficiencies for DOEs can be obtained with kinoforms that are constructed as surface relief gratings on some substrate.[1] However, in order to achieve a high efficiency, it is necessary to resort to complex fabrication processes that can provide the required accuracies for controlling the graded shape and depth of the surface grooves. Specifically, in a single process one photomask with variable optical density is exploited for controlling the etching rate of the substrate to form the desired graded relief gratings, or using multiple binary photomasks so the graded shape is approximated by multilevel binary steps.[1,2] Both fabrication processes rely mainly on etching techniques that are difficult to accurately control. As a result, the shape and depth of the grooves may differ from those desired, which would lead to a reduction of diffraction efficiency and poor repeatability of performance.

Previous researches have begun to investigate polarization diffraction gratings consisting of spatially rotating polarizers[3] or waveplates.[4] Recently we have demonstrated simple polarization diffraction gratings based on continuous space-variant computer-generated subwavelength gratings.[5] The derived phase was not a result of optical path differences, but solely due to local changes in polarization, which was in fact a manifestation of the geometrical space-domain Pancharatnam–Berry phase.[5–8] Optical elements that exploit this effect to form a desired phase front are called Pancharatnam–Berry phase optical elements (PBOEs).[5] However, applying the constraint on the continuity of the subwavelength grating leads to a space variation of the local period. As a result, the elements are restricted in their ability to form a desired complex phase function in addition to being limited in their physical dimensions. Moreover, the result of space varying periodicity complicates the optimization of the photolithographic process.

In this letter we present an approach for polarization dependent DOEs based on quantized Pancharatnam–Berry phase diffractive optics. We distinctly show that such elements can be realized with a discrete geometrical phase, using a computer-generated space-variant subwavelength dielectric grating. By discretely controlling the local orientation of such grating, which has uniform periodicity, we have the ability to form more complex and sophisticated phase elements. We experimentally demonstrate quantized Pancharatnam–Berry phase optical elements (quantized PBOEs) as a blazed diffraction grating and a polarization dependent focusing lens, for the 10.6 μm wavelength from a CO_2 laser. We show that high diffraction efficiencies can be attained by utilizing a single binary computer-generated mask, as well as forming multipurpose polarization dependent optical elements that are suitable for applications such as optical interconnects, polarization beam splitting, optical switching, and polarization state measurements.

The PBOEs are considered as wave plates with constant retardation and space varying fast axes, the orientation of which is denoted by $\theta(x,y)$. It is convenient to form such space varying wave plates using subwavelength grating. When the period of a subwavelength periodic structure is smaller than the incident wavelength, only the zero order is a propagating order, and all other orders are evanescent. The subwavelength periodic structure behaves as a uniaxial crystal with the optical axes parallel and perpedicular to the subwavelength grooves.[9] Therefore, by fabricating quasiperiodic subwavelength structures, for which the period and orientation of the subwavelength grooves were changed along the length of the element, we can realize space-variant waveplates.

It is convenient to describe PBOEs using Jones calculus. The space-dependent transmission matrix for the PBOE is given by applying the optical rotator matrix on the Jones matrix of the subwavelength dielectric grating to yield in helical basis[5]

$$\mathbf{T}(x,y) = \tfrac{1}{2}(t_x + t_y e^{i\phi})\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tfrac{1}{2}(t_x - t_y e^{i\phi})$$
$$\times \left\{ \begin{pmatrix} 0 & \exp[i2\theta(x,y)] \\ \exp[-i2\theta(x,y)] & 0 \end{pmatrix} \right\}, \quad (1)$$
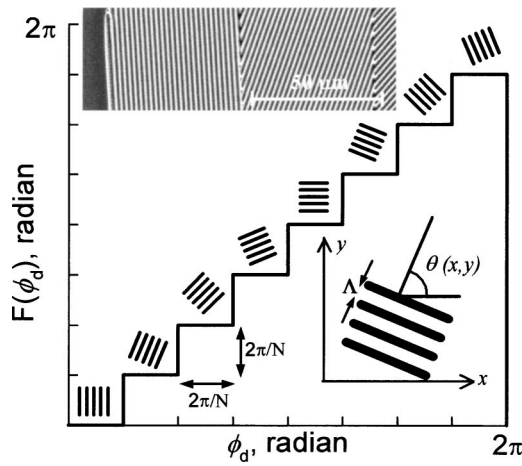
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FIG. 1. Actual quantized phase $F(\phi_d)$ as a function of the desired phase ϕ_d, as well as the discrete local grating orientation. Inset, a scanning electron microscope image of a region on the subwavelength structure of the focusing lens.
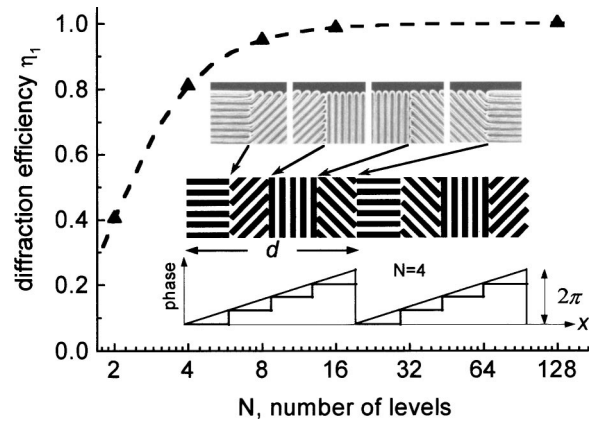

FIG. 2. The magnified geometry of the grating for $N=4$, as well as the predicted geometrical quantized phase distribution, and scanning electron microscopy images of some regions of the grating. Also shown, the measured (triangles) and predicted (dashed curve) diffraction efficiency as a function of the number of discrete levels, N.

where t_x, t_y are the real amplitude transmission coefficients for light polarized perpendicular and parallel to the optical axes, and ϕ is the retardation of the grating. Thus for an incident plane-wave with arbitrary polarization $|\mathbf{E}_{\text{in}}\rangle$, we find that the resulting field is

$$|\mathbf{E}_{\text{out}}\rangle = \sqrt{\eta_E}|\mathbf{E}_{\text{in}}\rangle + \sqrt{\eta_R}e^{i2\theta(x,y)}|\mathbf{R}\rangle + \sqrt{\eta_L}e^{-i2\theta(x,y)}|\mathbf{L}\rangle, \quad (2)$$

where $\eta_E = |\frac{1}{2}(t_x + t_y e^{i\phi})|^2$, $\eta_R = |\frac{1}{2}(t_x - t_y e^{i\phi})\langle\mathbf{E}_{\text{in}}|\mathbf{L}\rangle|^2$, $\eta_L = |\frac{1}{2}(t_x - t_y e^{i\phi})\langle\mathbf{E}_{\text{in}}|\mathbf{R}\rangle|^2$, are the polarization order coupling efficiencies, $\langle\alpha|\beta\rangle$ denotes inner product, and $|\mathbf{R}\rangle = (1\ 0)^T$ and $|\mathbf{L}\rangle = (0\ 1)^T$ represent the right-hand and left-hand circular polarization components, respectively. From Eq. (2) it is evident that the emerging beam from a PBOE comprises three polarization orders. The first maintains the original polarization state and phase of the incident beam, the second is right-hand circular polarized and has a phase modification of $2\theta(x,y)$, and the third has a polarization direction and a phase modification opposite to that of the former polarization order. Note that the polarization order coupling efficiencies depend on the groove shape and material, as well as on the polarization state of the incident beam. For the substantial case of $t_x = t_y = 1$, *and* $\phi = \pi$ an incident wave with $|\mathbf{R}\rangle$ polarization is subject to entire polarization state conversion and results in emerging field

$$|\mathbf{E}_{\text{out}}\rangle = e^{-i2\theta(x,y)}|\mathbf{L}\rangle. \quad (3)$$

An important feature of Eq. (3) is the phase factor $\phi_d(x,y)|_{\text{mod }2\pi} = -2\theta(x,y)|_{\text{mod }2\pi}$ that depends on the local orientation of the subwavelength grating. This dependence is geometrical in nature and originates solely from local changes in the polarization state of the emerging beam.[5,8]

In our approach, the continuous phase function $\phi_d(x,y)$ is approximated with discrete steps leading to the formation of a PBOE with discrete local grating orientation. In the scalar approximation, an incident wave front is multiplied by the phase function of the quantized phase element described by, $\exp[iF(\phi_d)]$, where ϕ_d is the desired phase and $F(\phi_d)$ is the actual quantized phase. The division of the desired phase ϕ_d to N equal steps is shown in Fig. 1, where the actual quantized phase $F(\phi_d)$ is given as a function of the desired

phase. The Fourier expansion of the actual phase front is given by $\exp[iF(\phi_d)] = \Sigma_l C_l \exp(il\phi_d)$, where C_l is the lth-order coefficient of the Fourier expansion. The diffraction efficiency, η_l, of the lth-diffracted order is given by $\eta_l = |C_l|^2$. Consequently, the diffraction efficiency η_1 for the first diffracted order for such an element is related to the number of discrete levels N by $\eta_1 = [(N/\pi)\sin(\pi/N)]^2$. This equation indicates that for 2, 4, 8, and 16 phase quantization levels, the diffraction efficiency will be 40.5%, 81.1%, 95.0%, and 98.7%, respectively. The creation of a quantized PBOE is done by discrete orientation of the local subwavelength grating as illustrated in Fig. 1.

The objective was to design a blazed polarization diffraction grating, i.e., a grating for which all the diffracted energy is in the first order, when the incident beam is $|\mathbf{R}\rangle$ polarized. We designed a quantized PBOE that acts as a diffraction grating by requiring that $\phi_d = (2\pi/d)x|_{\text{mod }2\pi}$, forming the quantized phase function $F(\phi_d)$ depicted in Fig. 1, where d is the period of the diffraction grating. In order to illustrate the effectiveness of our approach, we realized quantized diffraction gratings with various number of discrete levels, $N = 2, 4, 8, 16, 128$. The grating was fabricated for CO_2 laser radiation with a wavelength of $\lambda = 10.6\ \mu\text{m}$, with diffraction grating period $d = 2.5$ mm and subwavelength grating period $\Lambda = 2\ \mu\text{m}$. The dimensions of the elements were 30 mm $\times$ 3 mm and consisted of 12 grating periods. The magnified geometry of the grating for the case, $N = 4$ and the predicted geometrical quantized phase distribution are presented in Fig. 2. The elements were fabricated on 500-μm-thick GaAs wafers using a single binary mask, contact photolithography, and electron-cyclotron resonance etching with BCl_3 to nominal depth of 2.5 μm, resulting in measured values of retardation $\phi = 0.4\pi$, and $t_x = 0.88$, $t_y = 0.77$. These values are close to the theoretical predictions achieved using rigorous coupled wave analysis. After the fabrication, an antireflection coating was applied to the backside of the element. By combining such gratings in cascade, we obtained a grating with a retardation phase close to π. The insets in Fig. 2 show scanning electron microscopy images of some regions of the grating with a number of discrete levels, $N = 4$, which was fabricated.

Following the fabrication, the quantized PBOEs were illuminated with a right-hand circularly polarized beam, $|\mathbf{R}\rangle$, at 10.6 μm wavelength. We used the circular polarizer to transmit only the desired $|\mathbf{L}\rangle$ state and to eliminate the $|\mathbf{R}\rangle$ polarization order that appeared due to the insufficient etched depth of the grating. Figure 2 shows the measured and predicted diffraction efficiency for first diffracted order for the different quantized PBOEs. The efficiencies are normalized relative to the total transmitted intensity for each element. The measured diffraction efficiency for $N=16$ was 99% $\pm 1\%$ rather than the 98.7% of the theoretical value. The excellent agreement between the experimental results and the predicted efficiency confirms the expected quantized phase.

In addition, we formed a quantized Pancharatnam–Berry phase focusing element for a 10.6 μm wavelength, having a quantized spherical phase function of $F(\phi_d)=F[(2\pi/\lambda) \times(x^2+y^2+f^2)^{1/2}]$, with a 10 mm diameter, focal length $f=200$ mm, and a number of discrete levels $N=8$. Figure 3 illustrates the magnified geometry of a focusing lens based on a quantized PBOE with $N=4$, as well as the predicted quantized geometrical phase. A scanning electron microscope image of a region on the subwavelength structure, which we had fabricated, is shown in the inset of Fig. 1. A diffraction limited focused spot size for $|\mathbf{L}\rangle$ transmitted beam was measured, while illuminating the element with $|\mathbf{R}\rangle$ polarization state, and inserting a circular polarizer. The inset in Fig. 3 shows the image of the focused spot size as well as the measured and theoretically calculated cross section. The measured diffraction efficiency was 94.5% $\pm 1\%$ in agreement with the predicted value. The geometrical phase of the PBOE is polarization dependent, therefore, we experimentally confirmed that our element is a converging lens for incident $|\mathbf{R}\rangle$ state, and a diverging lens for incident $|\mathbf{L}\rangle$ state, as indicated by Eq. (2). For incident $|\mathbf{L}\rangle$ state the measured focal length was $f=-200$ mm as expected, whereas the measured diffraction efficiency was identical to the measured incident $|\mathbf{R}\rangle$ state. Moreover, it is possible to form a bifocal lens as a PBOE with a retardation phase of $\phi=\pi$ while illuminating with a linear polarization beam, and inserting a refractive lens following the PBOE. A trifocal lens can be created as a PBOE with a retardation phase of $\phi=\pi/2$ resulting in three distinct focuses for $|\mathbf{R}\rangle$, linear, and $|\mathbf{L}\rangle$ polarization states.

To conclude, we have demonstrated the formation of quantized Pancharatnam–Berry phase optical elements using computer-generated space-variant subwavelength dielectric grating. We have realized blazed diffraction gratings, as well as a polarization dependent focusing lens. The introduction
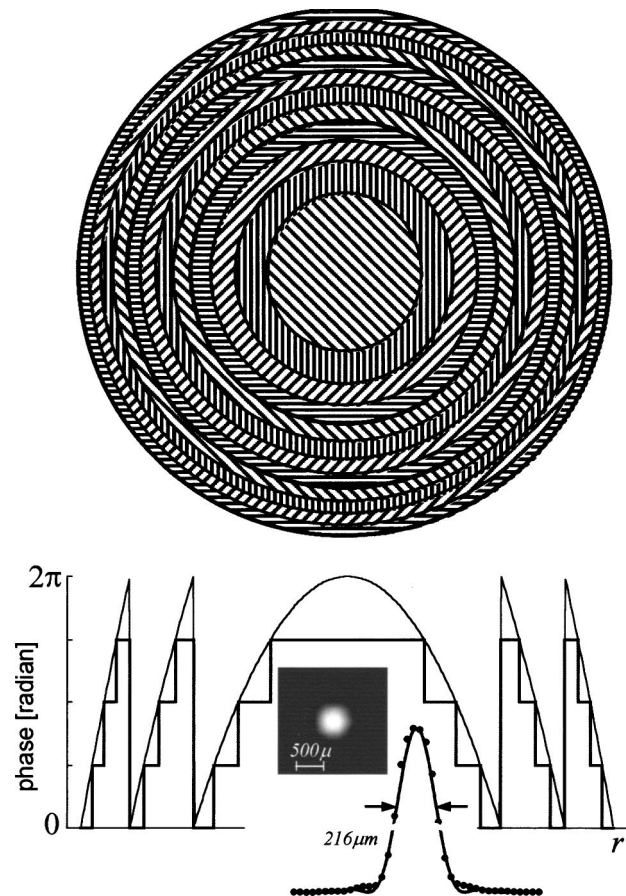

FIG. 3. Illustration of the magnified geometry of a quantized-PBOE focusing lens with $N=4$, as well as the predicted quantized geometrical phase. Inset; the image of the focused spot size as well as the measured (dots), and theoretically calculated (solid curve) cross section.

of space varying geometrical phases through quantized PBOEs, enables approaches for polarization-sensitive optical elements. We are currently investigating a photolithographic process with the purpose of achieving accurate control of the retardation phase to yield only the desired polarization order.

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