Computer-generated relief gratings as space-variant polarization elements

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A method for exploiting nonuniform ultrahigh spatial-frequency relief gratings as space-variant polarization elements is presented. In this method the local direction of the gratings determines the polarization angles, while the period of the gratings is controlled to ensure continuity of the grating function for any desired polarization operation. We illustrate the method with a specific space-variant half-wave plate for laser radiation of 10.6 μm.

Recent studies have shown that transmissive relief gratings of ultrahigh spatial frequencies can behave as homogeneous birefringent materials.[1–3] Such an artificial birefringence was exploited to form wave plates from several dielectric materials such as quartz,[1] polymethyl methacrylate,[2] and photoresist.[3] For these, only linear uniform gratings were recorded to form homogeneous space-invariant polarization elements. Sometimes, however, elements that have nonuniform, space-variant polarization are required. Possible examples are elements for transforming the azimuthal polarization of high-power annual CO_2 laser beams into a linear polarization and elements for polarization coding of the data in optical computing[4] and optical neural networks.[5] Such space-variant elements are difficult to produce with natural birefringent materials.

In this Letter we present a method for exploiting nonuniform relief gratings for space-variant polarization elements. We show that by controlling the local direction and period of the grating grooves, we can obtain the desired polarization change and continuity. Our method is then illustrated with a specific space-variant half-wave plate that transforms a wave front with uniform linear polarization into one with nonuniform polarization.

Consider a binary relief linear grating in air with a period much smaller than the illumination wavelength λ. The effective birefringence Δn for this grating can be derived in accordance with the form birefringence model[6] as

$$\begin{aligned}\Delta n &= n_{\parallel} - n_{\perp} \\ &= [q + (1-q)/n^2]^{-1/2} - [q + (1-q)n^2]^{1/2}, \quad (1)\end{aligned}$$

where $n_{\parallel}$ and $n_{\perp}$ are the effective refractive indices for light polarized parallel and perpendicular to the grating grooves, respectively, q is the form factor (linewidth/period), and n is the refractive index of the grating material. For a normally incident wave (in the z direction), the retardation phase Φ between these two orthogonal polarizations for a grating with relief depth t and effective birefringence Δn is

$$\Phi = (2\pi/\lambda)\Delta n t. \quad (2)$$

When the relief depth is chosen as $t = \lambda/(2\Delta n)$, the resulting retardation phase is $\Phi = \pi$ rad (180°), so the grating serves as a half-wave ($\lambda/2$) plate. Such a half-wave plate grating transforms an input wave with linear polarization in direction α_{in} into an output wave with linear polarization in a different direction α_{out}, where the direction of the grating grooves is $\beta = (\alpha_{\mathrm{in}} + \alpha_{\mathrm{out}})/2$ (all the angles are measured relative to the y axis). The grating vector is perpendicular to the grating grooves and may be written as

$$\mathbf{K} = K_0 \sin(\beta)\hat{\mathbf{y}} - K_0 \cos(\beta)\hat{\mathbf{x}}, \quad (3)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the x and y directions, and $K_0 = 2\pi/\Lambda$ denotes the spatial frequency of the grating, with Λ the grating period. Note that the effective birefringence properties of the grating do not depend on the period, as long as it is much smaller than the wavelength.[6] Consequently, within such constraints on the period, K_0 may be arbitrarily chosen. The relationship among α_{in}, α_{out}, β, and $\mathbf{K}$ is illustrated in Fig. 1.

Now let us assume that the direction of the linear polarizations of the input and output waves are no

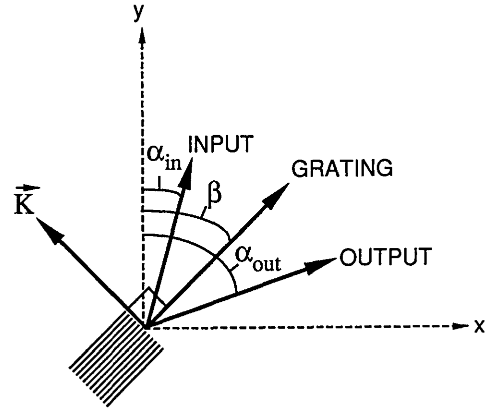

Fig. 1. Diagram illustrating the relationship among α_{in}, α_{out}, β, and $\mathbf{K}$.

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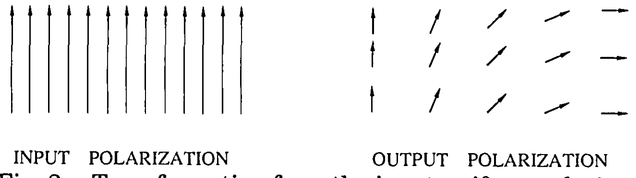

Fig. 2. Transformation from the input uniform polarization to the output nonuniform polarization. The polarization direction is indicated by the arrows.

longer constants but are space varying, i.e., $\alpha_{\mathrm{in}}(x,y)$ and $\alpha_{\mathrm{out}}(x,y)$. Here the polarization element may still be a half-wave plate grating, but now the direction of the grating grooves must be a function of the coordinates, given as

$$\beta(x,y) = [\alpha_{\mathrm{in}}(x,y) + \alpha_{\mathrm{out}}(x,y)]/2\,. \tag{4}$$

Thus the grating is no longer linear and is described by the following grating vector as

$$\mathbf{K}(x,y) = K_0(x,y)\sin[\beta(x,y)]\hat{\mathbf{y}} - K_0(x,y)\cos[\beta(x,y)]\hat{\mathbf{x}}\,, \tag{5}$$

where $K_0(x,y) = 2\pi/\Lambda(x,y)$ is now the local spatial frequency of the grating, which has yet to be determined.

For the grating vector of Eq. (5) to be physically realizable in a continuous way, it should be a conserving vector, i.e., it should fulfill the following relation[7]:

$$\nabla \times \mathbf{K} \equiv \frac{\partial K_x}{\partial y} - \frac{\partial K_y}{\partial x} = 0\,. \tag{6}$$

Incorporation of Eq. (5) into Eq. (6) yields

$$K_0\left(\cos\beta\,\frac{\partial\beta}{\partial x} - \sin\beta\,\frac{\partial\beta}{\partial y}\right) + \sin\beta\,\frac{\partial K_0}{\partial x} + \cos\beta\,\frac{\partial K_0}{\partial y} = 0\,. \tag{7}$$

Equation (7) is a necessary condition for $K_0(x,y)$ at which a continuous grating whose local groove direction is $\beta(x,y)$ exists. Once such a conserving grating vector is determined, we can readily derive the grating function $\phi(x,y)$ by integrating the grating vector along any arbitrary path in the xy plane.[7] For example, the path $(0,0) \to (x,0) \to (x,y)$ leads to the equation

$$\phi(x,y) = \frac{\lambda}{2\pi}\int_0^x K_x(x',y=0)\mathrm{d}x' + \frac{\lambda}{2\pi}\int_0^y K_y(x,y')\mathrm{d}y'\,. \tag{8}$$

Let us consider a simple example to illustrate the procedure described above. We assume an input wave with uniform linear polarization, say in the x direction $\alpha_{\mathrm{in}}(x,y) = 0$, and an output wave whose direction of polarization is a linear function of the x coordinate $\alpha_{\mathrm{out}}(x,y) = ax$, where a is a constant. Such a transformation is illustrated in Fig. 2, where the arrows denote the direction of the linear polarization. The local direction of the grating grooves is obtained from Eq. (4) to be $\beta(x,y) = a'x$, where $a' = a/2$, and the corresponding grating vector can be written as

$$\mathbf{K}(x,y) = K_0(y)\sin(a'x)\hat{\mathbf{y}} - K_0(y)\cos(a'x)\hat{\mathbf{x}}\,. \tag{9}$$

In Eq. (9), $K_0(y) = 2\pi/\Lambda(y)$ is assumed to depend only on the y coordinate. Incorporation of Eq. (9) into Eq. (7) yields the requirement on $K_0(y)$, which is

$$\frac{\mathrm{d}K_0(y)}{\mathrm{d}y} = -a'K_0(y)\,. \tag{10}$$

Equation (10) can be solved analytically to obtain

$$K_0(y) = \frac{2\pi}{\Lambda_0}\exp(-a'y)\,, \tag{11}$$

where Λ_0 is the maximal grating period (at $y = 0$) and must be smaller than λ. Finally, the grating function is obtained from Eqs. (8), (9), and (11) to be

$$\phi(x,y) = -\frac{\lambda}{\Lambda_0 a'}\sin(a'x)\exp(-a'y)\,. \tag{12}$$

We also performed a modest experiment that is based on our method. In this experiment we used the grating function from the first example [Eq. (12)] to record an element on gallium arsenide (GaAs) with $n = 3.27$ and tested it at 10.6-μm radiation from a CO_2 laser. The shape of the element was chosen as a rectangle, $0 \le x \le 15$ mm and $0 \le y \le 3$ mm, and the direction of the output polarization was chosen as $0° \le \alpha_{\mathrm{out}} \le 90°$. These choices led to a constant $a' = \pi/60\ \mathrm{mm}^{-1}$ in Eq. (12). Also, the maximal grating period was taken as $\Lambda_0 = 10\ \mu$m, slightly smaller than the illumination wavelength, and the resulting minimal grating period (at $y = 3$ mm) was approximately 8 μm. Finally, the form factor that we chose was $q = 1/2$, so the effective birefringence as obtained from Eq. (1) is $\Delta n = 1.065$. Thus, in order to obtain the retardation phase $\Phi = \pi$ of a half-wave plate, the relief depth of the grating grooves should be $t = 5.0\ \mu$m in accordance to Eq. (2).

A Lee-type[8] binary mask having a grating function of Eq. (12) was first plotted with a high-resolution

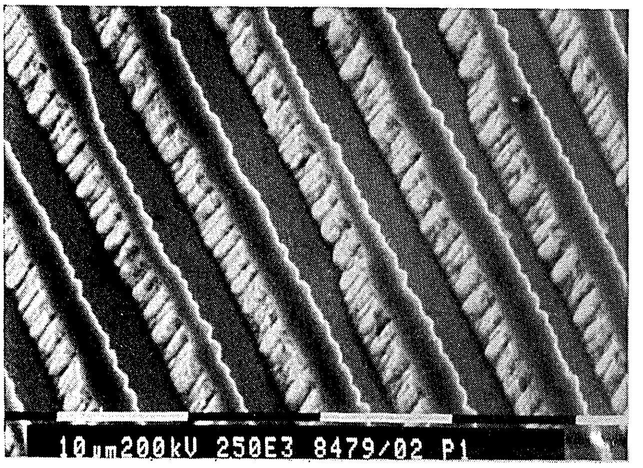

Fig. 3. Electron microscope picture of the relief pattern from a typical etched section of the polarization element.

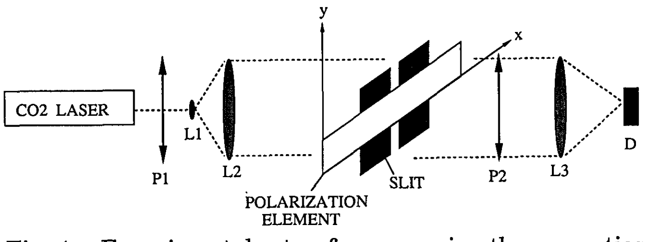

Fig. 4. Experimental setup for measuring the properties of the polarization element. P1, P2, polarizers; L1–L3, lenses; D, detector.

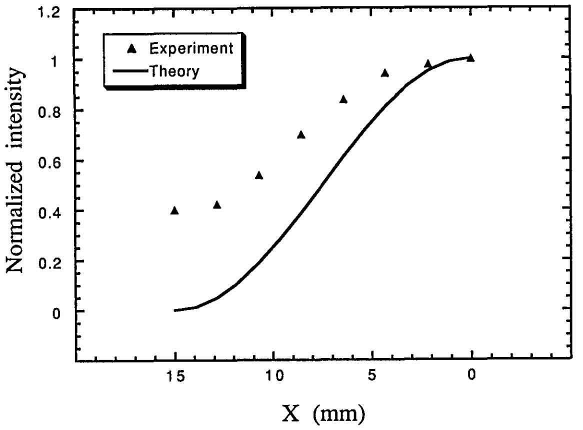

Fig. 5. Experimental and predicted normalized light intensities as a function of the position of the slit.

laser plotter, then optically demagnified by a factor of 10 onto a photographic plate, which was then used in a contact print to yield a chrome mask. The data from the chrome mask were transferred to the GaAs wafer by a standard lithographic process and wet etching with H_3PO_4:H_2O_2:H_2O (1:1:25).[9] The desired π retardation phase was obtained by combining two identical elements in cascade,[3] each with a relief depth of 2.5 μm. Figure 3 shows an electron microscope picture of the relief pattern from a typical etched section of the element.

The experimental setup is illustrated in Fig. 4. The polarization element is illuminated by a normally incident plane wave derived from the CO_2 laser and polarized in the y direction. The output wave from the element passes through a linear polarizer in the y direction and is collected by a lens to a detector. Finally, a 1-mm-wide slit is translated across the element (in the x direction) so as to measure the light intensity as a function of x. The measured normalized light intensity, together with the theoretically predicted intensity as a function of the slit translation x, is shown in Fig. 5; since the predicted direction of the output polarization of the element is $\alpha_{\text{out}} = ax$, the predicted intensity is simply $\cos^2(\alpha_{\text{out}}) = \cos^2(ax)$. As is evident, the experimental results have the same monotonic behavior as predicted, indicating that space-variant operation on the polarization was indeed achieved. However, there is some quantitative discrepancy between theory and experiment. Specifically, the measured intensity does not reach zero as predicted but remains at approximately 40% of the maximal value. This unpredicted light has approximately circular polarization.

Several possible explanations can be given to account for the discrepancies between the theoretical and experimental results in Fig. 5. First, the form birefringent theory that we used is accurate only when the grating period is significantly smaller than the wavelength.[6] In our experiment, owing to the limited resolution of the plotter and the lithographic process, the grating period was only slightly smaller than the wavelength, so deviations from the theory are possible; moreover, several diffraction orders indeed propagate inside the wafer (substrate modes), so the validity of the form birefringence model in this case is questionable. Second, experimental errors in the etch depth, groove shape, and form factor may have altered the retardation angle. Finally, because gratings with ultrahigh spatial frequency have some antireflection effects,[10] the transmittance of light polarized parallel and perpendicular to the grating lines ($T_\parallel$ and $T_\perp$) may differ, whereas in our calculations we assumed that the transmittance would be equal for both polarizations; note that the transmittance may be a function of both x and y coordinates.

To conclude, we demonstrated how ultrahigh spatial-frequency relief gratings can be exploited as space-variant polarization elements, where the local direction of the gratings determines the polarization angles, while the local period of the gratings ensures continuity. In our experiment, we used gratings with periods that were only slightly smaller than the illumination wavelength. This led to some discrepancies between our predictions that were based on the simplified form birefringence model and the experimental results. These discrepancies may be substantially reduced either by exploiting gratings with periods that are much smaller than the illumination wavelength or by resorting to rigorous electromagnetic calculations of gratings.

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