# Realization of perfect shuffle and inverse perfect shuffle transforms with holographic elements

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Techniques for implementing perfect shuffle and inverse perfect shuffle operations with the aid of a single holographic optical element are presented. The element is composed of subholographic lenses which operate on a different input area. For the inverse perfect shuffle operation, polarization coding is added in order to separate the input into distinct groups. Experimental results illustrating the effectiveness of the proposed techniques are given.

## Introduction

Free-space optical interconnects offer low cross talk, high bandwidth, and parallel operation, and are therefore attractive for use in digital optical computers. Two particularly useful interconnect schemes are based on the perfect shuffle (PS) transform and its inverse ($PS^{-1}$). Recently several approaches for optical implementation of the PS have been proposed.[1-5] These approaches include the use of conventional optics, such as lenses and prisms,[1-3,5] as well as diffractive optics,[4] and are based on either imaging[1,2] or interferometric[4] techniques. Here we present novel optical implementations of the PS and $PS^{-1}$ transforms; both utilize a single holographic optical element (HOE) and are therefore simple, lightweight, and compact.

## PS Transform

The PS transform can be regarded as a one-to-one permutation of a group of $N$ inputs. Specifically, it divides the group into two halves and then interlaces the inputs from one half into the second half. Mathematically, the PS is defined by the following expression[6]:

$$n' = \begin{cases} 2n & \text{if } 0 \le n < N/2 \\ 2n - N + 1 & \text{if } N/2 \le n < N \end{cases}, \tag{1}$$

where $n'$ is the new location of the $n$th input. The importance of the PS transform in parallel computer architecture may be attributed to the fact that it can be performed $3 \log_2 N$ times on a group of $N$ inputs so as to obtain an arbitrary permutation. After each PS transformation, neighboring pairs of elements are interchanged or not interchanged in a prescribed manner.[3] The PS transform may be extended to operate on two-dimensional arrays of inputs. For example, it can operate as a separable transform that is capable of performing a one-dimensional PS in each dimension.[2]

Optical implementation of a two-dimensional PS transform involves separating the input data into four quadrants, stretching each quadrant to the original size of the whole data, shifting each quadrant by the proper amount for interlacing, and then combining the four quadrants into a single image. These operations were performed with several prisms and lenses by Lohmann *et al.*[1,2] A similar approach by Brenner and Huang[3] suggests the use of beam splitters, lenses and slant mirrors. In our implementation, all these operations are performed by a single HOE.

The optical arrangement is schematically shown in Fig. 1. The input data, which is depicted by numbers, may be a transparency or a spatial light modulator. The HOE is composed of four off-axis subholographic spherical lenses, each forming an image of one quadrant from the input with the proper shift and proper magnification (2×). Finally the output is a reversed version of the desired shuffle. This reversal can be corrected, if necessary, with an imaging lens. For each of the four subholographic lenses we recorded an interferogram that results from the interference of a diverging spherical wave with a converging spherical wave. The diverging wave emerged from a pinhole that is located at the center of the corresponding quadrant in the input plane and the converging wave was focused by a high-quality lens to the center of the

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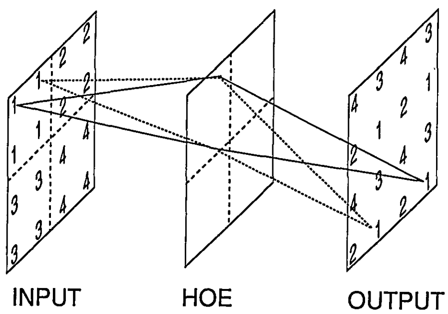



Fig. 1. Optical arrangement for implementing a two-dimensional PS. Only some of the rays that emerge from the input are shown.

output plane. This procedure ensures that proper interlacing of the four quadrants will take place. The HOE was recorded on a 10E56 Agfa photographic plate, which was then processed and bleached to obtain high diffraction efficiency. The illumination source was an argon laser ($\lambda$ = 514.5 nm), and computer-controlled translation and rotation stages were used to control the geometric parameters.

In our experiments, the input data was composed of an array of 8 × 8 numbers that were written on a transparency at intervals of 3 mm. The distance between the input plane and the HOE was 200 mm and the distance between the HOE and the output plane was 400 mm, giving the desired 2× magnification. Finally an off-axis angle of 20° was added to ensure separation of various diffraction orders. The input array and the corresponding output array are shown in Fig. 2. As can be seen, the proper interlacing of the quadrants is performed with high optical quality over the entire field of view.

### $PS^{-1}$ Transform

The $PS^{-1}$ transform can be defined mathematically by interchanging $n$ and $n'$ in Eq. (1). In order to implement the $PS^{-1}$ transform, the same kind of operations should be performed on the input data as for the PS transform: separating, stretching (here by 0.5× instead of 2×), shifting, and adding. The last three operations may be performed in a way that is similar to the PS realization. However, the separating operation is more problematic. In the PS transform the quadrants to be interlaced were initially spatially separated. Therefore it was possible to divide the HOE into subholograms, with each operating on a corresponding input area. However, in the $PS^{-1}$ the input data are already interlaced and if spatial separation is used, as it is for the PS, the size of each subhologram would be the size of a pixel. This, of course, would mean that the optical system is now space variant (unlike an imaging system that can be described by convolution with some point spread function), so the space–bandwidth product (SBP) that the system can handle will be reduced significantly.[7] A more direct explanation for the reduction in the SBP is that as the size of each subhologram decreases, the diffraction-limited spot size (pixel size) in the output plane increases. In order to retain a reasonable SBP, we developed a different method, which involves polarization coding, for separating the input data into groups.

Our arrangement for optically implementing a one-dimensional $PS^{-1}$ is shown in Fig. 3. The input is coded with an interlaced polarizing mask. As shown, the subareas (pixels) of the input are alternately covered with vertical and horizontal polarizers; the odd pixels are covered with vertical polarizers whereas the even pixels are covered with horizontal polarizers. The input is illuminated with diffuse laser light that is derived from an argon laser ($\lambda$ = 514.5 nm). The HOE now is composed of two subholograms. The first subhologram is covered with a vertical polarizer so as to transmit light coming from the odd pixels only, and the second subhologram is covered with a horizontal polarizer so as to transmit light coming from the even pixels only. The two subholograms were recorded in a similar way as the PS implementation, but now the distance between the input plane and the HOE is 400 mm, and the distance between the HOE and the output plane is 200 mm; this ensures the desired 0.5× demagnification. The input data, which are composed of a row of 8 numbers at 3-mm intervals, are shown in Fig. 4(a) along with the corresponding output [Fig. 4(b)]. As is evident, the interlaced output data are indeed separated, with low cross talk among the various numbers.

In order to implement optically a two-dimensional $PS^{-1}$, four additional degrees of freedom are needed

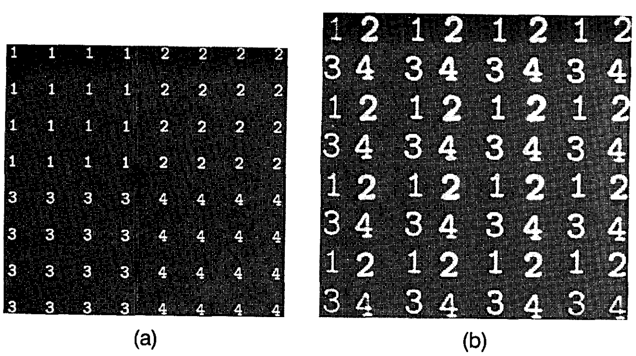



Fig. 2. Experimental results for an optical two-dimensional PS: (a) input array, (b) output array.

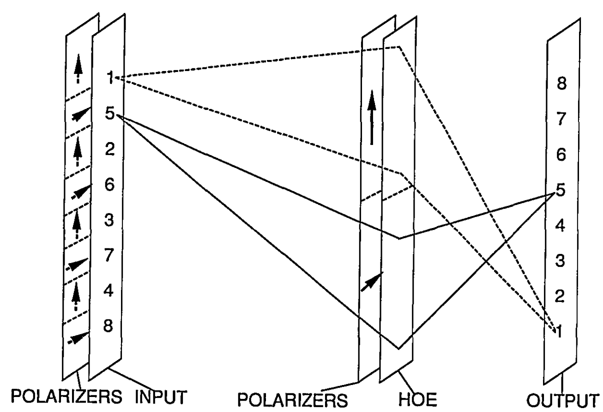
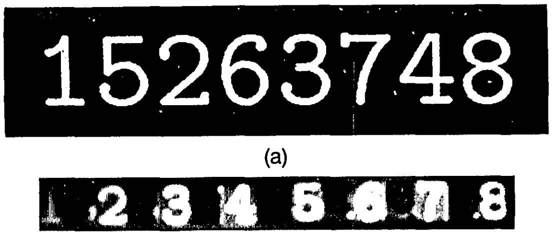



Fig. 3. Optical arrangement for implementing a one-dimensional $PS^{-1}$ transform.




(a)

(b)

Fig. 4. Experimental results for an optical one-dimensional $PS^{-1}$: (a) input array, (b) output array.

for separating the four independent quadrants of the input data. Here, polarization coding alone, with only two independent degrees of freedom, is not enough. A possible solution in this case may be to use four different wavelengths to illuminate the input data and corresponding filters on each pixel and each subhologram; actually the filters on the subholograms may be omitted by exploiting Bragg wavelength selectivity of volume holograms. It is also possible to combine two polarization states and two wavelengths for coding and separation.

### Concluding Remarks

We have demonstrated how to implement optically a two-dimensional PS transform and a one-dimensional $PS^{-1}$ transform, each with a single HOE. Such implementations lead to simple and compact arrangements that are based on shift-invariant operations and can therefore handle data with a high SBP.

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