# High-resolution spectrometry for diffuse light by use of anamorphic concentration

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A new scheme to improve the spectral resolution of grating-based spectrometers for diffuse light is proposed and demonstrated. It exploits an anamorphic transformation that reduces the beam divergence in the direction of the grating grooves while increasing the divergence in the orthogonal direction to improve the spectral resolution without any loss of light. Up to 12-fold improvement in the spectral resolution was obtained. © 1999 Optical Society of America

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Grating-based spectrometers require that one use a narrow (or well-collimated) input to ensure high spectral resolution. For diffuse light sources such as light bulbs, arc lamps, and the Sun, and for large thermal emitters, a narrow input slit is thus required that can decrease the input signals by many orders of magnitude. For totally incoherent radiation any improvement in the divergence of the beam impinging upon the grating without such substantial loss of power violates the optical brightness (or étendue) conservation theorem<sup>1</sup> and is therefore forbidden.

Here, we propose to reduce the beam divergence only in the direction of the grating grooves (i.e., the direction that specifies the spectral resolution) while increasing the divergence in the orthogonal direction. Such an anamorphic transformation conserves the total (two-dimensional) optical brightness but can still largely improve the spectral resolution without any loss of light. For the anamorphic transformation we propose and demonstrate a novel design that involves a one-dimensional prism array, together with two cylindrical lenses to perform the same operation as the much more complicated arrangements that were used before.<sup>2</sup> Since this design involves only reflective and refractive optical elements, it has  $\sim 100\%$  light efficiency and small chromatic aberrations and is very simple to construct and align. We demonstrate theoretically and experimentally large improvements in the spectral resolution, nearly without loss of light.

To analyze the spectral resolution of a gratingbased spectrometer for diffuse light we consider a diffuse light beam with size  $D_x$  by  $D_y$  and divergence angles  $\sin \alpha_x$  and  $\sin \alpha_y$  in the x and y directions, respectively. The beam quality in each direction can be characterized by  $M_x^2 = D_x \sin \alpha_x / \lambda$  and  $M_y^2 =$  $D_y \sin \alpha_y / \lambda$ , where  $\lambda$  is the central light wavelength. Note that  $M^4 = M_x^2 M_y^2 = D_x D_y \sin \alpha_x \sin \alpha_y / \lambda^2$  is consistent with the common notation for characterizing the optical quality of laser beams.<sup>3</sup>  $M_x^2$  and  $M_y^2$  are the ratios of the divergence angle to the diffractionlimited angular spread for each direction (for a TEM<sub>00</sub> Gaussian laser beam,  $M^4 = M_x^2 = M_y^2 = 1$ ). When such a diffuse beam is diffracted by a grating (say, in the *x* direction) and then focused by a lens, we obtain the wavelength resolution  $d\lambda$  by equalizing the grating's angular dispersion to the diffuse angle as

$$\mathrm{d}\lambda = (M_x^2/Nq)\lambda\,,\tag{1}$$

where  $N = D_x/\Lambda$  is the number of grating lines illuminated by the beam (where  $\Lambda$  is the grating period) and q is the diffraction order. Equation (1) is a direct generalization of the well-known relation for a diffractionlimited plane wave (with  $M_x^2 = 1$ ) and indicates that the spectral resolution for a low-quality beam is reduced by exactly  $M_x^2$ . Note that  $M_y^2$  does not affect the spectral resolution. Therefore an anamorphic transformation that substantially decreases  $M_x^2$  while conserving  $M^4$  can improve the spectral resolution without any loss of light.

Our optical setup for high-resolution spectrometry with diffuse light, illustrated in Fig. 1, consists of four blocks. Block 1 is used to prepare an effective



Fig. 1. Experimental optical arrangement for our highresolution spectrometer: 1-4, building blocks; D, diffuser;  $A_0$ ,  $A_1$ , apertures;  $L_0-L_4$ , lenses. The arrows mark the ray paths through the optical system in the x-z plane.

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source with well-controlled size and divergence angles. Block 2 is the anamorphic concentrator that reduces  $M_x^2$  and increases  $M_y^2$  while conserving  $M^4$ . Block 3 is a homemade grating-based spectrometer, and Block 4 is a commercial spectrometer that is used to measure the spectral resolution of the homemade spectrometer. We now describe each block of the optical setup in more detail. To simplify the notation we use the paraxial approximation ( $\alpha \approx \sin \alpha \approx \tan \alpha$ ).<sup>4</sup> We also resort only to geometrical optics, since for our experiments the diffraction-limited angles and spot sizes are much smaller than the diffusive ones.

The effective source is prepared with a white-light 75-W xenon arc lamp that illuminates a diffuser adjacent to a circular aperture,  $A_0$  (with diameter  $D_0 = 11 \text{ mm}$ ), both of which are located at the back focal plane of a collimating spherical lens,  $L_0$ , with a focal distance  $F_0 = 365 \text{ mm}$ . A second aperture,  $A_1$ , is adjacent to the lens and sets the beam size of the effective source to be  $D_{1x} = D_{1y} = 25 \text{ mm}$ . The source divergence (full) angles are  $\alpha_{1x} = \alpha_{1y} \cong D_0/F_0 = 0.03 \text{ rad}$ . Note that the effective source is isotropic, with beam quality factors  $M_{1x}^2 = M_{1y}^2 = 1185$  (for  $\lambda = 633 \text{ nm}$ ).

Block 2 in the optical setup in Fig. 1 is the anamorphic concentrator, which is operated in three simple stages: (i) focus in the x direction with cylindrical lens L<sub>1</sub>, with a focal distance  $F_1 = 80$  mm; (ii) exchange the divergence angles in the x and y directions with an array of one-dimensional Porro prisms; and (iii) focus again in the x direction with an additional cylindrical lens, L<sub>2</sub>, with a focal distance  $F_2 = 8$  mm. The principle of our anamorphic transformation is illustrated in Fig. 2. Figure 2(a) presents the effective source. The dimensions  $D_{1x} = D_{1y}$  are represented by the size of the rectangle, and the lengths of the double-shafted arrows represent the divergence angles  $\alpha_{1x} = \alpha_{1y}$ . We continue to use this graphic notation to represent the (four-dimensional) spatial and angular light distribution at subsequent cross sections [Figs. 2(b)-2(d)].

The light distribution at the focal plane of L<sub>1</sub> is presented in Fig. 2(b). Now the x direction is much smaller than the y direction but with a larger divergence angle  $(D_{2x} = F_1\alpha_{1x} = 2.4 \text{ mm} \ll D_{2y} \cong$  $25 \text{ mm}, \alpha_{2x} = D_{1x}/F_1 = 0.31 \text{ rad} \gg \alpha_{2y} = 0.03 \text{ rad}).$ Note that  $D_{2x}\alpha_{2x} = D_{2y}\alpha_{2y} = D_{1x}\alpha_{1x} = D_{1y}\alpha_{1y}$ , so  $M_{2x}^2 = M_{2y}^2 = M_{1x}^2 = M_{1y}^2$  as expected (here we neglect the small increase,  $F_1 \sin \alpha_{1y} = 2.4 \text{ mm}$ , in  $D_{2y}$ during free propagation for a distance  $F_1$ ).

Next the divergence angles in the x and y directions are interchanged by use of an array of one-dimensional retroreflectors located at the focal plane of L<sub>1</sub> and oriented at 45° to the x and y axes. Each retroreflector is a 1 mm by 50 mm 90°-reflecting Porro prism.<sup>5</sup> The light distribution immediately after reflection from the prism array is shown in Fig. 2(c). As can be seen from the figure, for each small prism both size and divergence are exchanged between the x and y directions. However, for the entire input, the x and y divergence are interchanged,  $\alpha_{3x} = \alpha_{2y}, \alpha_{3y} = \alpha_{2x}$ , whereas the total x and y sizes are nearly unchanged,  $D_{3x} \cong D_{2x}D_{3y} =$  $D_{2y}$ .<sup>6</sup> Therefore, after the light is reflected from the prism array, the x direction is much smaller than the y direction and also has much smaller divergence angles. Finally, the beam is concentrated again in the x direction with a second cylindrical lens,  $L_2$ , with focal distance  $F_2 = 8$  mm. At the focal plane of  $L_2$  the beam size is  $D_{4x} = F_2 \alpha_{3x} = 0.24$  mm,  $D_{4y} = D_{3y} = 25$  mm, and the divergence angles are  $\alpha_{4y} = \alpha_{3y} = 0.31$  rad and  $\alpha_{4x} = D_{3x}/F_2 = 0.30$  rad [Fig. 2(d)]. Figure 3 shows the measured x cross sections of the light distribution at that plane (dashed curve). Also shown is the x cross section of the light distribution at the focal plane of  $L_1$ . As can be seen from Fig. 3, the anamorphic concentrator reduced the beam size in the x



Fig. 2. Illustrations of spatial and angular light distributions at several planes along the optical axis of the anamorphic concentrator (block 2 of Fig. 1). The width and height of the rectangle represent the dimensions of the beam in the *x* and *y* directions, respectively, and the lengths of the double-shafted arrows represent the diffuse angles in each direction: (a) at the input, (b) at the back focal plane of  $L_1$ before retroreflection (the one-dimensional Porro prism array is also shown for orientation), (c) after retroreflection, (d) at the back focal plane of  $L_2$ .



Fig. 3. Measured light-intensity distributions at the output of the anamorphic concentrator (dashed curve) and at the back focal plain of  $L_1$  (solid curve).



Fig. 4. Measured spectral impulse response of the homemade spectrometer with (solid curve) the anamorphic concentrator and (dashed curve) without it. The FWHM resolution is 2.8 and 36 nm with and without the concentrator, respectively.

direction by a factor of 10, while maintaining the same divergence angle, thereby reducing  $M_x^2$  by a factor of 10 (from 1185 to 12,000). The size of y at both locations is still ~25 mm, but  $\alpha_y$  after anamorphic concentration is ~10 times larger, yielding an increase of ~10 times in  $M_y^2$  (from 1185 to 12,000) and the required conservation of  $M^4$ .

The third block in the optical setup in Fig. 1 is a homemade spectrometer composed of collimating spherical lens  $L_3$  (focal distance  $F_3 = 85$  mm), a grating with 850 lines/mm (operated in the first diffraction order), and a focusing spherical lens,  $L_4$  (with  $F_4 = 200$  mm). Since no input slit was included, no loss of light occurred. The fourth block consisted of a commercial spectrometer whose input slit was located at the focal plane of  $L_4$ . We adjusted the input slit of the commercial spectrometer to 50  $\mu$ m to obtain a resolution of 0.1 nm, measured with a He–Ne laser. By scanning the input slit in the *x* direction, we measured the spectral resolution of the homemade spectrometer at various wavelengths.

The spectrum near 633 nm is shown in Fig. 4, yielding a spectral resolution of 2.8 nm (FWHM). Similar spectral width was measured throughout the visible spectral range (400-700 nm). For comparison, we also measured the spectral resolution of our homemade spectrometer with the anamorphic concentrator (block 2 of Fig. 1 and lens  $L_3$ ) removed. The results, also shown in Fig. 4, yield a spectral resolution of 36.8 nm (FWHM), in agreement with the theoretical prediction of Eq. (1) of 36 nm. The anamorphic concentrator thus yielded a 12-fold improvement in the spectral resolution for the same diffuse light source.<sup>7</sup> The total losses added by our anamorphic transformation were  $\sim 30\%$  and resulted mainly from Fresnel reflections from the uncoated cylindrical lenses and also from imperfect reflection from the Porro prisms.

To conclude, we have proposed and demonstrated a new scheme to improve the spectral resolution of grating-based spectrometers for diffuse light by anamorphic concentration that can largely improve the beam quality in one spatial dimension (at the expense of the orthogonal dimension). Since the scheme is without losses, it can be used for much weaker diffuse light sources than can conventional spectrometers that use a narrow input slit for improved spectral resolution. With proper choice of two cylindrical lenses and of the prism array, nearly diffraction-limited resolution can be achieved. However, in that case care must be taken to suppress the aberrations of all the optical components and in particular those of the (usually singlet) cylindrical lenses.

Finally, our arrangement for anamorphic transformation of diffuse light beams is useful for many applications that require concentration of diffuse light in one lateral direction. Two examples are concentration of solar radiation into narrow water pipes<sup>8</sup> and formation of scanning light lines in optical scanners, faxes, and copy machines and for optical metrology.<sup>9</sup> Several techniques for anamorphic transformations by use of optical fibers,<sup>10</sup> diffractive optical elements,<sup>11,12</sup> arrays of micro-optical components,<sup>13,14</sup> and a twomirror stack<sup>15</sup> were recently proposed. However, all these transformations were used to concentrate linear diode laser bars into symmetrical spots, whereas our anamorphic transformation performs the inverse (or time-reversal) operation.

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- 4. For the largest angle used in our experiment, the paraxial approximation yields an error of <1%. In any case we can readily generalize the expression in the text to deal accurately with arbitrary large angles.
- 5. We also rotated the prism array at  $45^{\circ}$  to the optical axis to separate the reflected wave from the incident wave. This rotation results in simple folding of the optical axis, as shown in Fig. 1.
- 6. This is true when the retroreflector width W is much smaller than  $D_{2x}$ . Otherwise,  $D_{3x} \cong D_{2x} + W/2$ .
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