# Efficient formation of a high-quality beam from a pure high-order Hermite–Gaussian mode

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We present a relatively simple method for efficiently transforming a single high-order mode into a nearly Gaussian beam of much higher quality. The method is based on dividing the mode into equal parts that are then combined coherently. We illustrate the method by transforming a Hermite–Gaussian (1, 0) mode with $M_x^2 = 3$ into a nearly Gaussian beam with $M_x^2 = 1.045$. Experimental results are presented and compared with theoretical results. © 2002 Optical Society of America

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The quality of a laser beam is normally defined by the focusability of the beam, which is the product of beam waist and beam divergence. The focusability can be determined by the beam propagation factor, usually denoted $M^2$.[1] A beam with a Gaussian intensity distribution has an optimal beam quality of $M^2 = 1$. Such a beam is derived from a laser in which an aperture is inserted into the resonator such that the laser operates with the fundamental mode only. Unfortunately, in this case, only a small volume of the gain medium is exploited, leading to a significant reduction of the output power with respect to multimode operation.

To increase the output power and also to retain good beam quality we investigated lasers that operate with a single pure high-order mode.[2–4] To achieve single high-order mode operation we introduced into the laser resonator both binary and continuous phase elements. It was shown that binary phase elements cannot improve beam quality.[5] Alternatively, continuous phase elements can substantially improve $M^2$ of either a single[6] or several[7] high-order modes, but they do introduce some inherent losses. In all these cases the emerging beam quality is still lower than that of a Gaussian beam.

In principle, because of well-defined amplitude and phase distributions of a pure high-order mode,[8] the mode's transformation into a nearly Gaussian beam is allowed by thermodynamics.[6] Such a transformation can be performed by means of two specially designed external phase elements,[9] but design and formation of such elements are difficult, if not impossible.

In this Letter we present a relatively simple and tractable approach to efficiently transforming a high-order mode into a nearly Gaussian beam. This approach is based on the fact that the field distributions of high-order Laguerre–Gaussian and Hermite–Gaussian (HG) modes often consist of several bright spots separated by dark interfaces with zero fields. The adjacent spots normally have opposite phases ($\pi$ phase shift), and the intensity distribution of each spot is rather close to that of the Gaussian beam. For example, for each of the two spots of the $HG_{10}$ mode we calculate that $M_x^2 = 1.15$, nearly the same as that for the Gaussian beam; this value is much smaller than for the entire $HG_{10}$ mode with $M_x^2 = 3$. Our approach to mode transformation is to separate and then to add coherently the field distributions of the individual spots of a high-order mode. We illustrate our approach by transforming the intensity distribution of a single $HG_{10}$ mode derived from a cw Nd:YAG laser into a nearly Gaussian beam. We determine the conditions for the highest efficiency of such a transformation and calculate the resultant $M^2$.

A possible arrangement for separating the distribution of the $HG_{10}$ mode into two symmetric parts and then combining them coherently is shown in Fig. 1. A sharp edge of a mirror is carefully aligned along the symmetry axis between the two spots, such as

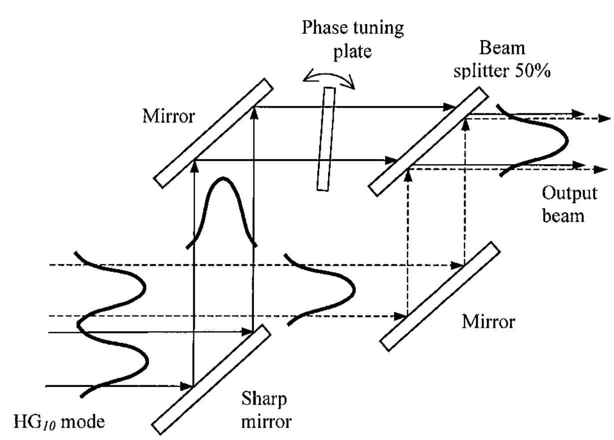



Fig. 1. Arrangement for obtaining a nearly Gaussian beam from a $HG_{10}$ mode. The sharp mirror reflects only one spot, whereas the other two mirrors reflect the spots toward the 50% beam splitter where they are combined coherently. The phase tuning plate permits fine adjustment of the relative phases of the two beams to produce the best coherent summation.

to reflect only one of them. Then the two beams are reflected by mirrors and combined with a 50% beam splitter, as in a Mach–Zehnder interferometer. A phase tuning plate is inserted in the path of one of the beams to adjust the phase between them by slight tilting of the plate. With the appropriate phase between the two beams, the resultant combined beam will emerge either to the right of the beam splitter or directed upward.

Even though the beams may be regarded as coming from different sources, they have a definite phase relationship, which allows them to be coherently superposed. Effectively, the field distribution of one spot of the $HG_{10}$ mode is shifted with respect to the other. This shift is illustrated in Fig. 2, where the dashed curves denote the field distributions of the two individual spots and the solid curve denotes their sum. One half of the mode is shifted by shift parameter $x_0/w$, where $x_0$ is in units of the waist $w$ of the HG mode ($w$ is a radius for which the Gaussian term falls to its $1/e$ value). As is evident, the two separated distributions cannot completely coincide, so there is some power leakage. Because the two distributions of the individual spots are combined coherently, it is best to have a maximal power $P$ in one direction, while the power leakage $\Delta P$ (the power in the other direction) will be minimal. The relative power leakage $\Delta P/P$ can be written as

$$\frac{\Delta P}{P} = \frac{\int_{-\infty}^{\infty}[U_1(x) - U_2(x - x_0)]^2 \mathrm{d}x}{2\int_{-\infty}^{\infty} U_1{}^2(x) + U_2{}^2(x - x_0)\mathrm{d}x}\,, \tag{1}$$

where $U_1(x)$ and $U_2(x - x_0)$ are the field distributions of the two spots of the $HG_{10}$ mode, which have the same phase. Beam quality factor $M^2$ of the combined beam along the $x$ direction, in accordance with Ref. 10, is

$$M_x{}^2 = 4\pi\sigma_x\sigma_{s_x}\,, \tag{2}$$

where $\sigma_x$ and $\sigma_{s_x}$ are the near-field and the far-field standard deviations of the beam intensity profile in the $x$ direction (spatial frequency $s_x$ is related to propagation angle $\theta$ by $s_x = \sin\theta/\lambda$).

Using Eqs. (1) and (2), we calculate the power leakage and $M_x{}^2$ of the combined beam as a function of relative shift parameter $x_0/w$. The results are presented in Fig. 3. Figure 3(a) shows the relative power leakage. It is evident that the power leakage is minimal at $x_0 = 1.6w$, where it is only 1.5%. Figure 3(b) shows the calculated $M_x{}^2$ factor of the combined beam. At the optimal value of $x_0 = 1.6w$, $M_x{}^2 = 1.045$, close to the diffraction limit and much smaller than $M_x{}^2 = 3$ for the original $HG_{10}$ mode. Surprisingly, it is even smaller than the $M_x{}^2$ factor of each spot separately (which is 1.15).[11] Obviously, we have a significant reduction of the value of $M_x{}^2$, from 3 for the $HG_{10}$ mode[1] to 1.045. Because the $M_y{}^2$ factor (in the $y$ direction) remains 1, the effective cylindrical $M^2$ value[12] will be $M^2 = (M_x{}^2 + M_y{}^2)/2 = 1.0225$, near that of an ideal Gaussian beam.

In our experiments we used the $HG_{10}$ mode emerging from a Nd:YAG cw laser that contained an intracavity discontinuous phase element.[4] The experimental near-field and far-field intensity distributions of the mode were measured with a CCD camera and are presented in Fig. 4. We obtained the far-field intensity distribution by focusing the output beam with a spherical lens ($f = 101$ cm). These results indicate that the experimental $HG_{10}$ mode is quite pure, so it should be possible to transform that mode into a high-quality nearly Gaussian beam. From the data of Fig. 4 we calculated that $M_x{}^2 = 3.21$ and $M_y{}^2 = 1.1$, slightly larger than the theoretical values $M_x{}^2 = 3$ and $M_y{}^2 = 1$.

The $HG_{10}$ mode was then introduced into the arrangement shown in Fig. 1. The mirrors and the beam splitter were carefully adjusted for optimal overlap of the spots in both the near field and the far field, and the phase tuning plate was rotated until complete constructive interference was obtained at the output. This procedure ensured that the directions of the two beams were completely matched and that there was no phase difference between the beams. We verified

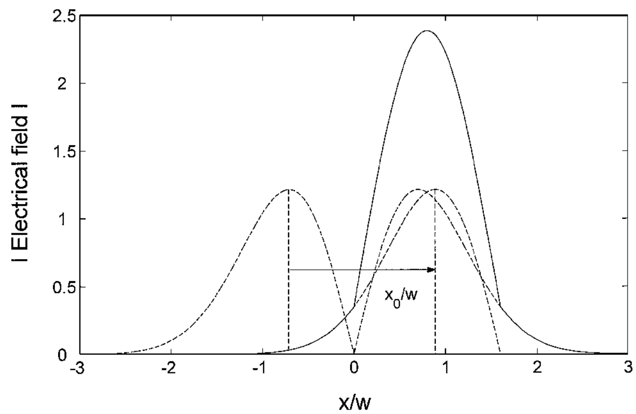



Fig. 2. Coherent summation of the two spots of the $HG_{10}$ mode. The left-hand spot (dashed curve) is shifted to the right-hand by a shift parameter $x_0/w = 1.6$ (see arrow), so it will almost coincide with the right-hand spot (dashed curve) and thereby produce the coherent sum of the two spots (solid curve).

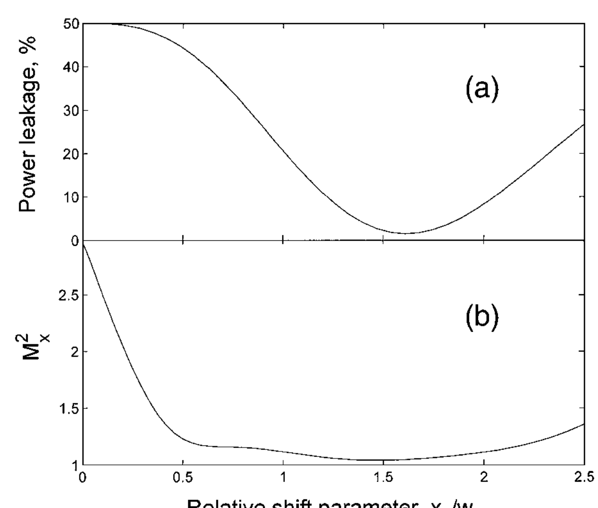
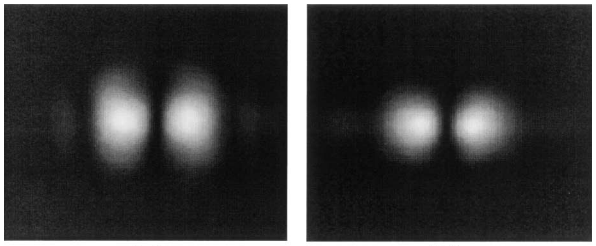
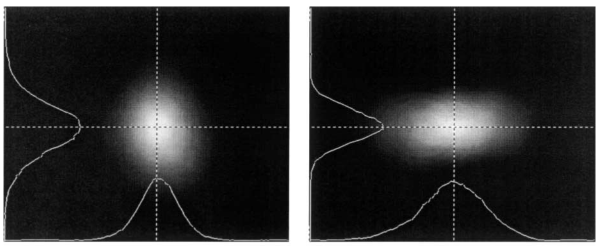



Fig. 3. (a) Relative power leakage and (b) beam-quality factor $M^2$ as functions of relative shift parameter $x_0/w$. It is evident that the power leakage is minimal at $x_0/w = 1.6$. The $M^2$ value obtained at $x_0/w = 1.6$ is 1.045.



(a) (b)

Fig. 4. Experimental intensity distributions of the $HG_{10}$ mode: (a) near field, (b) far field.



(a) (b)

Fig. 5. Experimental intensity distributions of the output high-quality, nearly Gaussian beam: (a) near field, (b) far field. Cross sections in the $x$ and $y$ directions are shown at the bottom and left-hand sides.

that the power leakage can be only a few percent, which indicates that the relative phase difference between the two beams can be almost completely suppressed. Note that we accomplished this suppression by using $\lambda/10$ optics, standard mirror mounts, and a nonfloating optical table. Moreover, the coherent summation should be made sufficiently close to the original beam waist to minimize wave-front curvature of either beam. In our experiment the entire optical distance from the output coupler was ~35 cm and the Rayleigh distance was ~3 m.

The experimental intensity distributions obtained in the near and far fields are shown in Fig. 5. Both have the expected shape of one bright spot, with nearly Gaussian cross sections in both the $x$ and the $y$ directions. Using these results, we calculated that ${M_x}^2 =$ 1.34 for the output beam, somewhat higher than the expected value of 1.045. We attribute this discrepancy to a possible impurity of the original mode. Still, there is reasonable agreement between the predicted and the experimental results, proving the validity of our approach.

To summarize, we have presented a novel and simple method for efficiently transforming a single high-order mode into a nearly Gaussian beam, thereby improving the beam quality. The method was verified experimentally with an $HG_{10}$ mode, with results close to the predicted ones. This method can easily be extended to other high-order modes, for example, the $HG_{11}$ mode, which is composed of four identical nearly Gaussian nodes and has ${M_x}^2 = {M_y}^2 = 3$.[12] In this case the arrangement of Fig. 1 can be applied twice to fold the mode in two perpendicular directions.

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