## Efficient multilevel phase holograms for CO<sub>2</sub> lasers

## E. Hasman, N. Davidson, and A. A Friesem

Department of Electronics, Weizmann Institute of Science, Rehovot 76100, Israel

Received October 25, 1990; accepted January 17, 1991

Multilevel phase holograms for monochromatic radiation at a wavelength of 10.6  $\mu$ m are recorded as surface relief gratings with multilevel discrete binary steps. Our experiments show that diffraction efficiencies close to 90% can be achieved both for transmissive and reflective elements. The reduction of efficiency due to errors in the depth and the width of the step levels is considered.

Holographic elements for  $10.6 - \mu m$  radiation are generally recorded as surface relief gratings on some substrate. For transmissive elements, the substrate must be transparent to  $10.6 - \mu m$  radiation (e.g., gallium arsenide, zinc selenide, or germanium), whereas for reflective elements the relief pattern can be overcoated with a reflective layer so the substrate need not be transparent. The diffraction efficiency of such holographic elements depends on the shape of the gratings. For example, the efficiency of kinoforms, which have continuously graded surface relief gratings, can reach 100%.<sup>1,2</sup> In practice, however, the kinoforms must generally be approximated with multilevel discrete binary phase steps.<sup>3,4</sup> To ensure that high diffraction efficiencies are obtained, the errors due to the depth and width of the step levels must be taken into account. In this Letter we report our investigations with multilevel phase holograms, in which we evaluate the reduction of diffraction efficiency due to these errors. We also record and test holographic focusing lenses for the 10.6- $\mu$ m wavelength from a CO<sub>2</sub> laser and show that high diffraction efficiencies can be reached.

The diffraction efficiency for the multilevel phase holographic optical elements can be calculated by using the scalar approximation. Note that the scalar assumption is valid only for a thin grating, where the criterion for determining whether the scalar approximation is valid is based on the parameter Q, given by<sup>2</sup>

$$Q = \frac{2\pi\lambda T}{n\Lambda^2},\tag{1}$$

where  $\lambda$  is the wavelength, *n* is the refractive index of the substrate,  $\Lambda$  is the local grating period, and  $T = \lambda / \Delta n$  is the optimal relief thickness, with  $\Delta n$  being the relief-modulating refractive-index change for transmissive elements and  $\Delta n = 2$  for reflective elements. When  $Q \ll 1$  the scalar approximation is valid; otherwise the diffraction efficiency should be solved directly from the basic Maxwell equations.<sup>5</sup>

In the scalar approximation, an incident wave front is multiplied by the phase function of the multilevel phase holographic optical element that is described by

$$H = \exp[iF(\phi)],\tag{2}$$

where  $\phi$  is the desired hologram phase and  $F(\phi)$  is the

actual quantized phase. The Fourier expansion of Eq. (2) is given by

$$\exp[iF(\phi)] = \sum_{l=-\infty}^{\infty} C_l \exp(il\phi), \qquad (3)$$

where  $C_l$  is the *l*th-order coefficient of the Fourier expansion, given by

$$C_l = \frac{1}{2\pi} \int_0^{2\pi} \exp[iF(\phi) - il\phi] \mathrm{d}\phi.$$
(4)

The diffraction efficiency,  $\eta_l$ , of the *l*th diffracted order is given by

$$\eta_l = \frac{|C_l|^2}{\sum_{k=-\infty}^{\infty} |C_k|^2},$$
(5)

where, for a pure phase hologram,  $\sum_{k=-\infty}^{\infty} |C_k|^2 = 1$ .

The division of the desired phase  $\phi$  to N equal steps is shown in Fig. 1, where the actual quantized phase  $F(\phi)$  is given as a function of the desired phase  $\phi$ . Solving Eq. (4) for the relevant first diffracted order (l = 1) and substituting the quantized phase  $F(\phi)$  from Fig. 1, we have the diffraction efficiency for the first diffracted order as a function of the number of levels N, where



Fig. 1. Actual quantized phase  $F(\phi)$  as a function of the desired hologram phase  $\phi$ .

© 1991 Optical Society of America



Fig. 2. Diffraction efficiency  $\eta_1$  as a function of the relative etch depth error  $\delta_d$  for N = 2, 4, 8, 16, and infinity step levels.

$$\eta_1 = |C_1|^2 = \left[\frac{N}{\pi} \sin\left(\frac{\pi}{N}\right)\right]^2. \tag{6}$$

Equation (6) indicates that for 2, 4, 8, and 16 phase quantization levels the diffraction efficiency will be 40.5%, 81.1%, 95.0%, and 98.7%, respectively.<sup>4</sup>

The creation of a multilevel phase hologram is done by multilevel lithography, where the hologram surface needs only to be etched m times in order to obtain  $N = 2^m$  levels. A different mask is used for each step, with a desired depth of each etch being

$$\Delta_m = \frac{\lambda}{\Delta n 2^m}.\tag{7}$$

We now consider how the depth errors, due to improper level etching, and the width errors, due to misalignment of the masks, affect the diffraction efficiency. First, we assume that the relative etch depth error  $\delta_d$  is equal for all the etch levels; this assumption represents the worst case for all possible combinations of relative errors less than or equal to  $\delta_d$ . With this assumption the diffraction efficiency as a function of the relative depth errors and the number of levels N becomes

$$\eta_1 = \left| \frac{1}{2\pi} \left[ \exp\left(-i\frac{2\pi}{N}\right) - 1 \right] \left[ \frac{1 - \exp(-i2\pi\delta_d)}{1 - \exp\left(-i\frac{2\pi}{N}\delta_d\right)} \right] \right|^2.$$
(8)

Figure 2 shows the efficiency as a function of the relative etch depth error  $\delta_d$  for N = 2, 4, 8, 16, and infinity. As shown, there is only a slight reduction of diffraction efficiency when the depth error is less than 10%. However, as the error increases beyond 10%, the reduction in efficiency becomes significant. For example, at 16 levels and 25% etch depth error the efficiency reduces to approximately 80% instead of the 98.7% with no error.

The effects of misalignment errors on the diffraction efficiency of kinoforms were investigated in detail.<sup>6</sup> We used a simplified approach in which we consider elemental areas of the hologram as multilevel phase gratings. The reduction of the diffraction efficiency as a function of horizontal misalignment relative to the minimal width of the step levels,  $\delta_w$ , was calculated numerically with Eq. (4). Here the misalignment of the *k*th step level was chosen to be  $(-1)^k \delta_w$ , a choice that gave the lowest diffraction efficiency among many distributions of misalignment smaller or equal to  $\delta_w$  that were considered. The results for this distribution (the worst case) with N = 2, 4, 8, and 16 levels are shown in Fig. 3. Again, relative misalignment of less than 10% does not cause any significant reduction in the diffraction efficiencies.

We recorded a multilevel holographic focusing element for a  $10.6-\mu m$  wavelength that had a spherical grating function with a 15-mm diameter and a 150-mm focal length. The desired relief pattern was obtained by multilevel lithography with the use of several



Fig. 3. Diffraction efficiency  $\eta_1$  as a function of the relative step width error  $\delta_w$  for N = 2, 4, 8, and 16 step levels.



Fig. 4. Surface profilometer traces for typical etched sections: (a) 8 levels of the reflective element, (b) 16 levels of the transmissive element.



Fig. 5. Relative power of the light as a function of the displacement of the knife edge (solid curve) and the corresponding intensity distribution (dashed curve).

masks. Each mask was first plotted as a binary computer-generated hologram, with a laser scanner (Scitex Raystar, Response 300) that had a resolution capability of approximately 10  $\mu$ m, and was then recorded onto a photographic film. The plots were demagnified optically and recorded as chrome master masks. The information from each mask was then transferred by contact printing and suitable exposure onto a single-crystal GaAs wafer, coated with an approximately  $1-\mu m$  photoresist layer (Shipley Microposit S1400-27). After the photoresist was developed, the GaAs was etched with H<sub>3</sub>PO<sub>4</sub>:H<sub>2</sub>O<sub>2</sub>:H<sub>2</sub>O (1:1:25), with an etch rate of 5.5 nm/s at 25°C, and then the remaining photoresist was removed. The etch depth of the mth mask was determined according to Eq. (7). A mask aligner with a resolution of approximately 1  $\mu$ m was used to align each mask onto the GaAs wafer. We used a semi-insulating GaAs wafer 450  $\mu$ m thick, with a crystallographic orientation of (100) and a refractive index of n = 3.27.

The maximal value of the parameter Q was calculated according to Eq. (1), with a minimal grating period for our specific focusing element of  $\Lambda_{\min} \simeq 212 \,\mu\text{m}$  and an optimal relief thickness of  $T = 4.67 \,\mu\text{m}$ . The result is  $Q = 2 \times 10^{-3}$ , which indicates that the scalar assumption for a thin grating is valid.

We recorded a reflective focusing lens with 8 levels and a transmissive focusing lens with 16 levels. For the reflective element, the etched GaAs wafer was overcoated with a thin gold layer of 0.1  $\mu$ m, whereas for

the transmissive element both the etch surface and the back planar surface were overcoated with antireflection layers. Figure 4 shows surface profilometer traces for typical sections of the two elements. Figure 4(a) depicts the 8 levels of the reflective element, and Fig. 4(b) depicts the 16 levels of the transmissive ele-The measured diffraction efficiency for the ment. reflective element was  $88 \pm 1\%$  rather than the theoretical value of 95%, whereas for the transmissive element the measured diffraction efficiency was  $87 \pm 1\%$ rather than the 98.7% of the theoretical value. We attribute the loss in efficiency to improper depths and misalignment errors as well as to insufficient loss reduction by the antireflection coating in the transmissive lens.

The focused spot sizes for both the transmissive and reflective lenses were measured with the scanning knife-edge method.<sup>7</sup> The results for the transmissive lens are shown in Fig. 5. The relative power at the focus plane as a function of the displacement of the knife edge is depicted by the solid curve. The intensity distribution was found by taking the derivative of the solid curve, and the result is shown by the dashed curve. The results indicate that the spot size is 260  $\mu$ m, which is the expected diffraction-limited size for these lenses.

To conclude, we have demonstrated that holographic elements can be recorded with multilevel lithographic techniques to obtain high diffraction efficiency. Although the experimental diffraction efficiencies are lower than those predicted by theory, they are sufficiently high to be useful for many applications. Further improvement of the diffraction efficiency should be possible by optimization of the photolithographic process and the antireflection coatings.

## References

- 1. W. H. Lee, in *Progress in Optics*, E. Wolf, ed. (Elsevier, Amsterdam, 1978), Vol. 16, pp. 119–232.
- R. Magnusson and T. K. Gaylord, J. Opt. Soc. Am. 68, 806 (1978).
- L. d'Auria, J. P. Huignard, A. M. Roy, and E. Spitz, Opt. Commun. 5, 232 (1972).
- G. J. Swanson and W. B. Veldkamp, Opt. Eng. 28, 605 (1989).
- M. G. Moharam and T. K. Gaylord, J. Opt. Soc. Am. 72, 1383 (1982).
- M. W. Farn and J. W. Goodman, Proc. Soc. Photo-Opt. Instrum. Eng. 1211, 125 (1990).
- E. Hasman, N. Davidson, A. A. Friesem, M. Nagler, and R. Cohen, Meas. Sci. Technol. 1, 59 (1990).