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Manipulating the Wigner distribution of high order laser modes

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Abstract

Despite the fact that the phase distribution across a beam emerging from a laser operating with several transverse modes is random, it is possible to improve the beam quality. This is achieved by manipulating the Wigner distribution function of the emerging beam, utilizing phase elements and a beam converter. The theoretical analysis along with experimental results with a Nd:YAG laser operating with two Laguerre–Gaussian modes are presented. © 2001 Elsevier Science B.V. All rights reserved.

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The quality of the beam emerging from a laser is usually defined by its focusability, determined by the product of the beam waist and beam divergence. A typical measure for the beam quality is the second order moment beam propagation factor, namely, the M^2 value [1,2]. Consequently, the optimal beam has a Gaussian shape, with a minimal waist and divergence product, which is limited only by diffraction. Such an optimal beam can be obtained from a laser operating with the fundamental mode. When the laser operates with multimodes, i.e., fundamental and higher order modes, the emerging beam quality is relatively poor and is mainly determined by the highest order mode. [1] In such lasers the phase distribution of the output beam is random, and little, if anything can be done to improve the quality of the beam. When the laser operates with a single high order mode, the emerging beam quality is still inferior to that from a laser operating with the fundamental mode, because the intensity distribution and the divergence of the beam are relatively large. Yet, a beam which originates from a laser operating with a single high order mode has well defined amplitude and phase

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[1] Slightly lower M^2 values are obtained for equipartitioned power between all modes. For example, there are 10 modes with $M^2 \leqslant 4$, and for equipartitioned power an overall value of $M^2 = 3$ is obtained. Consequently, the number of modes N_T is higher for equipartitioned power (e.g., for $M^2 = 4$, $N_T \sim 16$, and not 10).

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distributions, so it can be efficiently transformed into a nearly Gaussian beam [3]. This was supported by the analysis of the beam quality in accordance to entropy [4], which is minimal for beams with well defined amplitude and phase distributions.

Rather than operating with a single high order mode, a laser may also operate with only a few modes (where most of the modes between the fundamental and the highest order modes are not present). Here again the phase distribution is undefined at any point of the emerging beam. Yet, we will show both theoretically and experimentally, that it is possible to improve the quality of such beams. In the theory we will resort to Wigner algebra, particularly the Wigner distribution function (WDF), which is a useful tool for analyzing beam propagation and multi-mode systems [5,6]. Here, the WDF gives an intuitive representation of both the beam parameters and the optical beam transformation.

We begin by considering a laser with cylindrical symmetry, operating with Laguerre–Gaussian TEM_{pl} modes. The field distribution $E(r,\theta)$ is expressed by

$$E(r,\theta) = E_0 \varrho^{|l|/2} L_p^{|l|}(\varrho)\exp(-\varrho/2)\exp(\mathrm{i}l\theta), \qquad (1)$$

where r and θ are the cylindrical coordinates, E_0 the magnitude of the field, $\varrho = 2r^2/w^2$ with w as the spot size of the Gaussian beam, and L_p^l are the generalized Laguerre polynomials of order p and index l. For such a mode, the beam quality measure is [7] $M^2 = 1 + 2p + |l|$. For a laser operating with multi-modes and a certain M^2 value, the number of total modes is given by $N_{\mathrm{T}} = M^2(M^2+1)/2$. [1] With such a laser it is impossible to improve the beam quality. However, the beam quality could be improved in a laser operating with a limited number of modes N, much smaller than N_{T}, namely

$$N \ll N_{\mathrm{T}} = \frac{M^2(M^2+1)}{2} \approx \frac{\max(1+2p+|l|)^2}{2}. \qquad (2)$$

In order to illustrate how the quality of a beam emerging from a laser operating with few modes, could be improved, we resort to Wigner algebra. We start with the WDF, given by

$$W(\mathbf{r},\mathbf{p}) = \lambda^{-2}\int \mathrm{d}^2\Delta\mathbf{r}\exp(-2\pi\mathrm{i}\mathbf{p}\cdot\Delta\mathbf{r}/\lambda) \times \psi\left(\mathbf{r}+\frac{1}{2}\Delta\mathbf{r}\right)\psi^*\left(\mathbf{r}-\frac{1}{2}\Delta\mathbf{r}\right), \qquad (3)$$

where $\mathbf{r} = (x,y)$ is the spatial coordinate, $\mathbf{p} = (p_x,p_y)$ is the frequency coordinate and ψ denotes the field distribution. Fourfold sum expressions for the WDF of Laguerre–Gaussian beams were obtained [8], whereas elegantly simplified closed-form expressions yielded [9]

$$W_{p,\pm l}(\mathbf{r},\mathbf{p}) = \frac{2}{\lambda}(-1)^{2p+|l|}L_{p+|l|}(4[Q_0 \pm Q_2]) \times L_p(4[Q_0 \mp Q_2])\exp(-4Q_0), \qquad (4)$$

where $Q_0 = \frac{1}{2}(\mathbf{r}^2/w^2 + \pi^2 w^2\mathbf{p}^2/\lambda^2)$, $Q_2 = \pi\lambda^{-1}(xp_y - yp_x)$ and L_p are the Laguerre polynomials of order p. For the TEM_{00} fundamental mode we simply obtain a four-dimensional Gaussian. For all other modes, we obtain a wider and more complicated function, but still with a Gaussian envelope.

Two important properties of the laser output beam can be derived from the WDF. One is the M^2 value, which is the second order moment of the WDF [5], and corresponds to the four-dimensional volume surrounded by the envelope of the WDF. The other is the total coherence function K of a beam, given by [10]

$$K = \lambda\int W(\mathbf{r},\mathbf{p})^2\,\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{p} = \lambda\,\mathrm{tr}(W\cdot W^t). \qquad (5)$$

The total coherence function of Eq. (5) is inversely proportional to the actual (net) volume of the WDF, which may be only a part of the envelope volume. For lasers operating with the fundamental Gaussian mode or multi-modes, the actual and envelope volumes coincide, whereby the envelope is completely full. However, for lasers operating with either a single high order mode, or few modes, the envelope volume is significantly larger than the actual volume. Moreover, for lasers operating with any single high order mode, the total coherence function K is unity, just as for a laser operating with the fundamental Gaussian mode. This implies that the actual volume of the

WDF is minimal. For lasers operating with a few modes (each of equal power), the actual volume of the WDF is the number of modes. Thus, according to the Wigner algebra, it may be possible to reduce the WDF envelope volume of a beam from a laser operating with few modes, where condition of Eq. (2) is valid, towards that of the actual volume, thereby achieving the desired transformation. Here we present, for the first time, a practical and efficient method to improve the quality of beams emerging from lasers operating with a few modes. We illustrate it with an example of a laser operating with only two mutually incoherent transverse modes $TEM_{1,\pm1}$, whose M^2 value is 4. For comparison, the number of modes in a multi-mode laser with $M^2 = 4$ would be 10 (see Eq. (2)), thus, we indeed operate it with a few modes. [1]

The basic configuration of a laser resonator operating with two high order Laguerre–Gaussian modes, and beam converter arrangement for transforming the output beam into a nearly doughnut beam is shown in Fig. 1. Here, the mode selection is performed by inserting an annular discontinuous phase element (DPE) into the resonator, adjacent to the output coupler [11]. Moreover, the same DPE significantly improves the efficiency of the beam converter, since it changes the phase of the emerging beam. Thus, the DPE serves both as a mode selecting element and as a part of the beam converter.

For our calculations and experiments, we used a linearly polarized CW Nd:YAG laser in which the DPE consisted of a single discontinuity ring, with a specified radius, so as to simultaneously select both $TEM_{1,\pm1}$ modes. These two selected modes have the same radial dependence, but a different azimuthal dependence, thus, even an incoherent combination of these modes can be manipulated together in the radial direction. The optical beam converter contained a telescope configuration of two lenses, and a spatial filter in the form of a circular aperture. The intensity distributions next to the output coupler, at the spatial filter plane and at the output plane were detected with a CCD camera.

Since the DPE is placed next to the output coupler, the emerging field distribution does not have the usual π phase shift between the two rings, so the far-field intensity distribution is significantly affected. It now has a central ring which contains most of the output power and ring-shaped sidelobes that contain only a small portion of the power (e.g., 20.4% for the $TEM_{1,\pm1}$ modes). Thus, by exploiting the additional beam converter with a simple spatial filter (e.g., a circular aperture), it is possible to obtain a significant improvement in the M^2 value. Specifically, we can obtain a nearly doughnut beam, having a value of M^2 near 2 (theoretically 2.04 for the $TEM_{1,\pm1}$ modes), with only a small decrease (20.4%) in output power.

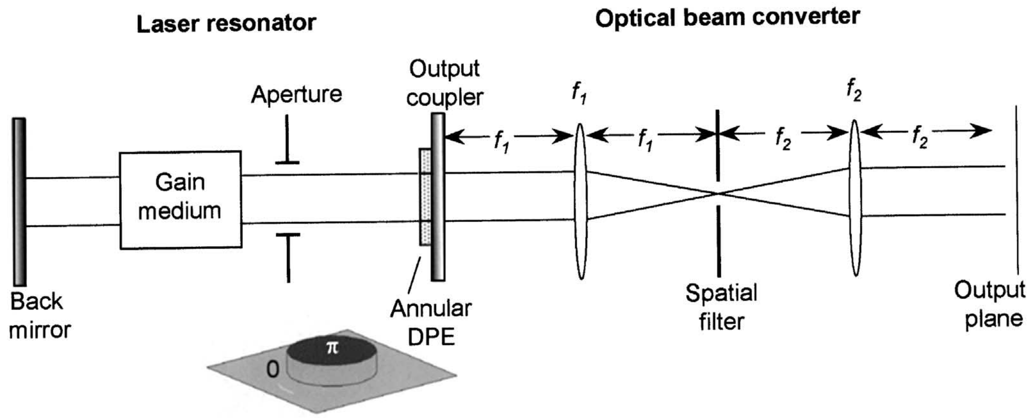

Fig. 1. Basic configuration of a laser resonator for obtaining high order $TEM_{1,\pm1}$ modes and optical beam converter for obtaining a nearly doughnut-shaped beam.

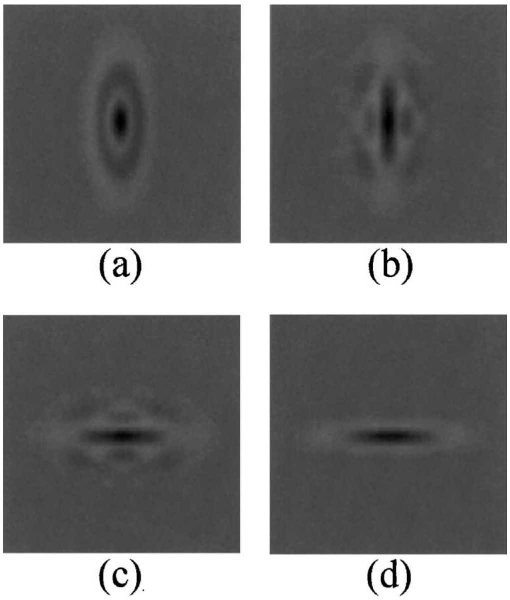

Fig. 2. WDF of the $TEM_{1,+1}$ mode at various locations (a) original $TEM_{1,+1}$ mode; (b) $TEM_{1,+1}$ mode after passing an annular DPE; (c) far field of $TEM_{1,+1}$ mode after passing an annular DPE; (d) $TEM_{1,+1}$ mode after passing an annular DPE and spatial filtering. All distributions are identical to those of the $TEM_{1,-1}$ mode and to an incoherent summation of the two modes.

Such a reduction in the M^2 value leads to a significant increase in the brightness of the beam, typically proportional to $P/(M^2)^2$, where P is the power, leading to an improvement by a factor of 3.1.

We calculated the WDF of the $TEM_{1,+1}$ mode, and some representative results are shown in Fig. 2. Since the Wigner distribution is four dimensional, we present only a sub-space which includes the origin and where **r** is parallel to **p**. Fig. 2(a) shows the WDF of a $TEM_{1,+1}$ mode, which consists of a central negative distribution, indicating a central phase singularity (vortex), and surrounding concentric rings. Fig. 2(b) shows the WDF after passing through a DPE which eliminates the π phase shift between the two rings in the field distribution of the $TEM_{1,+1}$ mode. As evident, the shapes of the rings not whole, leading to a high (and larger) central negative distribution, and more spread-out side lobes. This is the first stage in contracting the WDF. Fig. 2(c) shows the far-field WDF, which is obtained after Fourier transformation. This Fourier transformation practically rotates the WDF by 90°, to exchange the **r** and **p** axes. The last stage consists of a spatial filtering, which cleans the WDF from most of the side lobes, yielding a WDF similar to that of a $TEM_{0,+1}$ mode, still with a central negative distribution, but only a single outer ring. This WDF is shown in Fig. 2(d). Since the two modes $TEM_{1,+1}$ and $TEM_{1,-1}$ have a similar radial dependence (though a different azimuthal dependence), the above subspace of the WDF is the same for both modes, so Fig. 2(a)–(d) actually shows the WDF of either one of the two modes or of an incoherent superposition of the two. [2] Note that for only one of these modes, it is possible to further improve the beam quality with a spiral phase element. With the spiral phase element a uniform phase distribution would be obtained, leading to a nearly Gaussian output beam [3]. This spiral phase element has an azimuthal dependence, so it is beneficial only for one of these two modes, and degrades the beam quality of the other mode.

The results are presented in Figs. 3 and 4. Fig. 3 shows the detected intensity distributions, along with calculated and experimental cross-sections in the near and far fields respectively. Fig. 3(a) shows the intensity distribution and cross-sections at the output from the laser. Here we note the two rings shaped distributions of the $TEM_{1,\pm1}$ modes. The actual M^2 value of the beam inside the resonator was estimated using this measured intensity profile and the expected phase, to be close to the expected value of $M^2 = 4$. Fig. 3(b) shows the far-field intensity distribution and cross-sections. As evident, there is a central doughnut shaped pattern with some low side lobes, that are later removed by the spatial filtering, to obtain a nearly doughnut beam.

Fig. 4 shows the near- and far-fields intensity distributions along with calculated and experimental cross-sections of the beam at the output of the optical beam converter. As predicted, the intensity distribution is doughnut shaped in both the

[2] The WDF of an incoherent summation of two modes is the sum of the two WDFs of these modes.

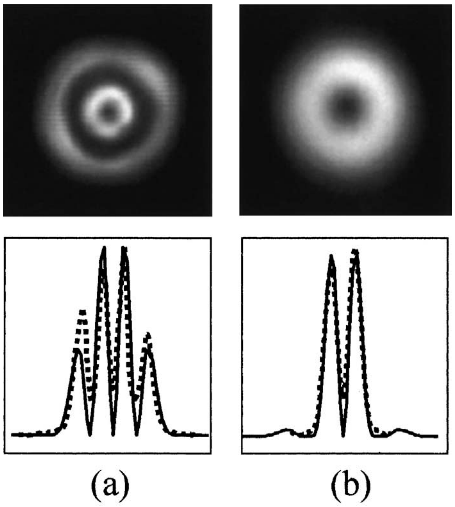

Fig. 3. Detected intensity distributions and (solid lines) calculated and (dashed lines) experimental cross-sections at near and far fields (a) at the output of the laser and (b) at the spatial filter plane.

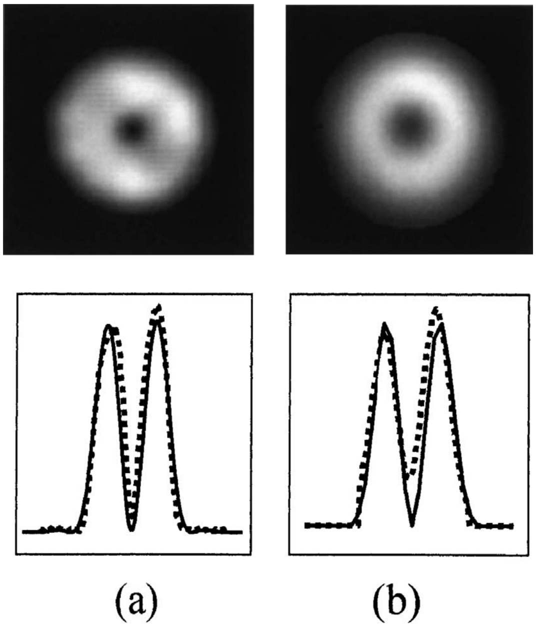

Fig. 4. Detected intensity distributions and (solid lines) calculated and (dashed lines) experimental cross-sections after of the optical mode converter (a) near field; (b) far field.

near and far fields. In this case the measured efficiency was 76%, which is in excellent agreement with the calculated limit of 79.6%. The M^2 value of this beam was nearly 2, as expected, leading to a significant brightness improvement. The M^2 values of the beam before and after the spatial filtering were obtained by the second order moments of the near- and far-field distributions (Figs. 3 and 4), yielding $M^2 = 4$ and $M^2 = 2$ before and after the spatial filtering respectively, up to $\sim$10% measurement accuracy. These M^2 values agree with predictions.

The laser operated with an output power of 9.5 W, resulting in 7.2 W after the mode converter. With a similar laser, with no phase element, an output power of 5.5 W was obtained by simply opening the aperture to allow the $TEM_{0,\pm1}$ modes operation. However, in this case, the laser operation included the fundamental mode, which was not suppressed. Thus, with our laser configuration we obtained a high-quality doughnut beam, with a significantly higher power (an increase of about 30%) than would normally be obtained.

To summarize, when the WDF of several high order modes have identical projection over some sub-space, the WDF can be simultaneously manipulated to reduce its actual volume, and hence to improve the beam quality. The final beam quality depends on the initial number of modes, and not necessarily on the highest order mode as would be in accordance to the M^2 criterion. We conclude that the M^2 criterion, which is very useful for lasers operating with either the single fundamental mode or multi-modes, may be inappropriate for lasers operating with only a few transverse modes. We noted that the transformation efficiency of our method was 79.6%. Such efficiency could be increased to 100% by resorting to spiral phase elements, rather than DPEs, and operating the laser with high order Laguerre–Gaussian modes having different p but the same l, such as $TEM_{0,+2}$ and $TEM_{1,+2}$.

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