E. Wolf, Progress in Optics 47 © 2005 Elsevier B.V. All rights reserved

### **Chapter 4**

### **Space-variant polarization manipulation**

by

### Erez Hasman, Gabriel Biener, Avi Niv, Vladimir Kleiner

Optical Engineering Laboratory, Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

### Contents

	Page
§ 1. Introduction	. 217
§ 2. Formation of space-variant polarization-state manipulations	. 222
§ 3. Geometrical phase in space-variant polarization-state manipulation	. 239
§ 4. Applications of space-variant polarization manipulation	. 260
§ 5. Concluding remarks	. 284
References	. 285

### §1. Introduction

One of the important properties of an optical wavefront is the transverse field distribution of the beam, that is, the distribution of its amplitude and phase as well as its polarization state. This field distribution determines the propagation behavior of the beam, and its angular momentum. The ability to generate an arbitrary complex scalar optical wavefront accurately is essential in modern optical applications. Although the ability to generate scalar beams is useful, in an increasing number of cases it is desirable to create arbitrary vectorial beams. In general, vectorial beams are defined as beams having space-variant (transversely nonuniform) polarization state.

Polarization is a fundamental property of electromagnetic fields. Accordingly, the state of polarization of light has substantial influence in most optical experiments and in the theoretical models developed to interpret them. In optics the polarization state of a light field can significantly affect the propagation of many fully or partially coherent paraxial light fields. However, its influence can never be ignored in nonparaxial conditions when the field propagates in free space. Significant polarization effects can occur during interaction with material interfaces such as gratings, or arrangements of nanoparticles. Comprehensive background information on polarization optics can be found in textbooks (see, for example, Collett [2003] and Brosseau [1998]).

Recent years have witnessed a growing interest, theoretically as well as experimentally, in space-variant polarization-state manipulation that can be exploited in a variety of applications. These include polarization encoding of data (Javidi and Nomura [2000]), neural networks and optical computing (Davidson, Friesem and Hasman [1992a]), optical encryption (Mogensen and Glückstad [2000]), tight focusing (Quabis, Dorn, Eberler, Glöckl and Leuchs [2000]), imaging polarimetry (Nordin, Meier, Deguzman and Jones [1999]), material processing (Niziev and Nesterov [1999]), and atom trapping and optical tweezers (Liu, Cline and He [1999]).

The study of polarization manipulation has grown into a new branch of modern physical optics known as polarization singularities (see, for example, Nye [1999] and Soskin and Vasnetsov [2001]). In a scalar field, such singularities appear at points or lines where the phase or the amplitude of the wave is undefined or changes abruptly. One class of such dislocations is formed by vortices, which are spiral phase ramps about a singularity. Vortices are characterized by a topological charge  $l = \frac{1}{2}\pi \oint \nabla \varphi \, ds$ , where  $\varphi$  is the phase of the beam and *l* is an integer. Until recently, research had focused mainly on field dislocations in scalar waves. However, if we allow for the polarization to be space-varying, disclinations can arise (see, for example, Nye [1983], Dennis [2002], and Freund, Mokhun, Soskin, Angelsky and Mokhun [2002]). Disclinations are points or lines of singularity in the pattern or direction of a transverse field. An example is the center of a beam with radial or azimuthal polarization.

Different techniques for obtaining space-variant polarization manipulation by use of nonuniform anisotropic polarization elements have been reported in the literature. In general, a polarization optical element is any optical element that can modify the state of polarization of a light beam, such as a polarizer, retarder, rotator, or depolarizer. Space-variant polarization elements can be implemented as space-variant computer-generated sub-wavelength dielectric or metal gratings (Hasman, Bomzon, Niv, Biener and Kleiner [2002]), polarization-sensitive materials such as azobenzene-containing materials (Todorov, Nikolova and Tomova [1984]), and liquid-crystal devices (Davis, McNamara, Cottrell and Sonehara [2000]). For the most general case, the transmission, retardation and optical-axis orientation of such elements depend on the location across the face of the element.

In order to analyze the beam emerging from a space-variant polarization element, we must resort to Jones and Mueller polarization-transfer matrix methods. The Jones calculus assumes completely polarized light and coherent addition of waves, whereas the Mueller calculus assumes partially polarized incoherent addition of waves. The space-dependent transfer matrices of Jones and Mueller can be calculated by expressing the local behavior of the element as a polarizer and retarder, while the local orientation of the optical axis can be obtained by applying the rotation matrix (see, for example, Collett [2003]). Moreover, Gori, Santarsiero, Vicalvi, Borghi and Guattari [1998] introduced an important method for analyzing partially coherent sources with space-varying partial polarization utilizing the beam coherence-polarization matrix (BCP). This approach can be viewed as an approximate form of Wolf's general tensorial theory of coherence (Wolf [1954]). Sometimes the local transmission and retardation of the polarization element cannot be determined in a straightforward manner, for instance when using gratings for which the period is close to or smaller than the incident beam's wavelength. In this case, a direct solution of Maxwell's equations is required. This can usually be accomplished by using numerical approaches, such as rigorous coupled wave analysis (RCWA; see Moharam and Gaylord [1986] and Lalanne and Morris [1996]), or utilizing finite-difference time-domain methods (FDTD) for analyzing nonuniform polarization optics (see, for example, Mirotznik, Prather, Mait, Beck, Shi and Gao [2000] and Jiang and Nordin [2000]).

Complex vectorial fields can be produced either by utilizing polarization elements (see, for example, Bomzon, Biener, Kleiner and Hasman [2002b]) or by using interferometric techniques involving two orthogonally polarized beams (see, for example, Tidwell, Ford and Kimura [1990]). A coherent summation, inside the laser resonator, of two orthogonally polarized TEM<sub>01</sub> modes was demonstrated by Oron, Blit, Davidson, Friesem, Bomzon and Hasman [2000]. Several designing approaches for obtaining vectorial fields having space-varying polarization distribution have been presented (see, for example, Niv, Biener, Kleiner and Hasman [2004] and Tervo, Kettunen, Honkanen and Turunen [2003]).

In a remarkable paper first published in 1956, Pancharatnam [1956] (reprinted in Pancharatnam [1975]) considered the phase of a beam of light whose polarization state is modified. Pancharatnam showed that a cyclic change in the state of polarization of the light is accompanied by a phase shift determined by the geometry of the cycle as represented on the Poincaré sphere (Brosseau [1998]). Therefore, space-variant polarization-state manipulations are accompanied by a phase modification that results from the Pancharatnam–Berry phase (Berry [1987], Bomzon, Kleiner and Hasman [2001d]). In order to investigate the propagation behavior of complex vectorial fields as well as the angular momentum (Allen, Padgett and Babiker [1999]), it is necessary to consider the resulting geometrical phase distribution. The calculation of the space-variant Pancharatnam phase is based on the rule proposed by Pancharatnam [1956] for comparing the phases of two light beams in different states of polarization as the argument of the vectorial projection between the two polarization states.

The propagation of paraxial vector fields has been extensively studied theoretically. Several vectorial treatments have been presented in both coherent (see, for example, Gori [2001]) and partially coherent light fields (see, for example, James [1994] and Seshadri [1999]). The simplest approach to studying arbitrarily polarized beams is to decompose the representative field vector at any point of a section into orthogonal linearly or circularly polarized parts. Free-propagation problems can then be performed as the analysis of the propagation of a pair of scalar waves (Goodman [1996]). On the other hand, for specific beams with axial-symmetric polarization distribution, significant results have been obtained by representing the field at a typical point as a superposition of radial and azimuthal components. Jordan and Hall [1994] showed that the propagation process of an azimuthally polarized Bessel–Gauss beam can be analyzed by means of a single one-dimensional propagation integral with a suitable kernel. A general vectorial decomposition

[4, § 1

of electromagnetic fields with application to propagation-invariant and rotating fields was presented by Pääkkönen, Tervo, Vahimaa, Turunen and Gori [2002]. Vectorial Talbot self-imaging and vectorial nondiffracting beams have been investigated for a wave field with periodic variations of the polarization state (e.g., Mishra [1991]), and have been demonstrated experimentally (Arrizón, Tepichin, Ortiz-Gutierrez and Lohmann [1996]). Moreover, Gori, Santarsiero, Borghi and Piquero [2000] introduced an important method for analyzing the propagation of partially coherent beams with space-varying partial polarization by extension of the van Cittert–Zernike theorem using the beam coherence polarization matrix (see, for example, Mandel and Wolf [1995]).

In the nonparaxial case, e.g., the propagation of the beam emerging from a lens with high numerical aperture (NA), one must resort to a vectorial formulation that takes into account polarization effects and nonuniformity of the amplitude over the wavefront. A mathematically tractable representation for dealing with polarization was developed by Debye [1909], and a representation for handling apodization was addressed by Hopkins [1943]. These developments were later generalized by Wolf [1959], applied to the analysis of aplanatic refractive lenses (free of spherical aberrations), and then exploited in investigations of the focal distribution in a variety of focusing systems (see, for example, Richards and Wolf [1959], Barakat [1987] and Sheppard and Wilson [1982]). Moreover, in a series of seminal papers Quabis and colleagues (Quabis, Dorn, Eberler, Glöckl and Leuchs [2001] and Dorn, Quabis and Leuchs [2003]) demonstrated both theoretically and experimentally the effectiveness of radially polarized doughnut beams focused by a high-NA lens in achieving significantly tighter focusing in far-field optics than had been possible with linearly polarized beams. The vectorial analysis of such propagation problems can be performed by using the generalized Debye integral, expressed for radially symmetric illumination (see, for example, Davidson and Bokor [2004]).

The optical properties of the vectorial beam can be evaluated by measuring the polarization distribution of the waves as well as their amplitude and phase distribution. The polarization state of the beam can conveniently be described geometrically by the polarization ellipse. In this case, a specific polarization state is characterized by the ellipticity and azimuthal angle of the major axis of an ellipse with respect to some reference frame (see, for example, Chapter 3.1 of Brosseau [1998]). The ellipticity and azimuthal angle can be calculated from the Stokes polarization parameters of the beam. There are four Stokes parameters, and these can be used to determine an intensity formulation of a beam's polarization state (Stokes [1852]). Therefore, they are directly accessible as linear combinations of the intensities measured by transmitting the beam through four different combina-

tions of optical elements referred to as wave plates and polarizers (see Chapter 5 of Collett [2003]). Henceforth, we will refer to this method as the four-measurements technique. Another useful method is to measure the time-dependent signal after the beam has been transmitted through a rotating optical component (referred to as a quarter-wave plate) and then through a polarizer (see Chapter 13 of Collett [2003]). In this case, the Stokes parameters are derived by Fourier analysis of the detected signal. Many versions of this method have been devised; Jellison [1987], for example, proposed the use of a photoelastic modulator instead of the rotating quarter-wave plate.

It is also desirable to be able to measure the transmission matrices of an optical element for either Jones or Mueller formalism. Many optical schemes have been proposed for this purpose. See Jones [1948], Raab [1982] and Brosseau [1985] for examples of Jones matrix measurement, and Lu and Chipman [1998] and Anderson and Barakat [1994] for demonstrations of Mueller matrix measurement. All these methods are based on a set of experiments in which known polarization states are fed into the optical system and the corresponding polarization states of the output beam are then measured.

In this chapter, theoretical analysis as well as experimental methods for obtaining space-variant polarization-state manipulation are reviewed along with several related applications. Various vectorial fields having space-variant polarization distributions are discussed in detail, together with the data from experimental studies.

The structure of this review is as follows: in Section 2 we review various methods for designing and realizing space-variant polarization-state manipulations. The use of sub-wavelength gratings, polarization interference methods and liquidcrystal devices for this purpose are considered. We also briefly describe the use of polarization-sensitive recording materials and discuss some general design approaches for space-variant polarization optics.

In Section 3 we consider optical phase elements based on the space domain Pancharatnam–Berry phase. Unlike with diffractive and refractive elements, the phase is not introduced through optical path differences, but results from the geometrical phase that accompanies space-variant polarization manipulation. Optical elements that use this effect to form a desired phase front are called Pancharatnam–Berry phase optical elements (PBOEs). The ability of PBOEs to generate complex wavefronts is demonstrated by forming helical wavefronts and polarization-sensitive beam splitters as well as polarization-dependent focusing lenses. The effect of the Pancharatnam–Berry phase on the propagation of vectorial beams is investigated in this study through the formation of either one- or two-dimensional vectorial propagation-invariant beams and vectorial Talbot effect. Section 4 elaborates on selected applications of space-variant polarization manipulation. Among the topics discussed are: near and far-field polarimetry, light depolarization, polarization encryption, optical computing, and spatial control over polarization-dependent emissivity. Finally, Section 5 presents some concluding remarks.

### §2. Formation of space-variant polarization-state manipulations

In this section we review the basic formation methods enabling the design and realization of space-variant polarization-state manipulations. We begin by describing the use of sub-wavelength gratings as space-variant polarizers and wave plates. A design method for performing two-dimensional polarization elements utilizing space-variant sub-wavelength gratings is presented. The method is based on determining the local direction and period of the sub-wavelength metal or dielectric gratings to obtain any desired continuous polarization change. As an example, we introduce the formation of linearly polarized axially symmetric beams with various polarization order numbers. We proceed by describing the spacevariant vectorial fields that are obtained by the polarization interference method. In this case vectorial fields are formed by the superposition of orthogonally polarized transverse modes. The superposition is carried out either externally, i.e., by using an interferometer, or within a laser cavity. Next, we proceed by describing space-variant polarization-state manipulations that are obtained using liquidcrystal devices. In this case the liquid-crystal device acts as a space-variant wave plate. Finally, we briefly describe the use of polarization-sensitive recording materials and some general design approaches for space-variant polarization-state manipulations.

# 2.1. Space-variant polarization-state manipulation by use of sub-wavelength gratings

Sub-wavelength gratings have opened up new methods for forming beams with sophisticated phase and polarization distributions. Such gratings are usually used to form homogeneous space-invariant polarizers or wave plates. When the period of the grating is much smaller than the incident wavelength, only the zeroth order is propagating, and all other orders are evanescent. These gratings behave as layers of a uniaxial crystal. Therefore, the use of space-variant (transversely inhomogeneous) sub-wavelength gratings permits the generation of complex vectorial wavefronts with a different polarization state at each location.

### 2.1.1. Background

The term "sub-wavelength gratings" refers to optical elements comprising typical structures that are smaller than the wavelength for which the elements were designed. Sub-wavelength gratings are usually one- or two-dimensional periodic structures, such as the one-dimensional grating depicted in fig. 1. When light is incident upon a sub-wavelength grating, all diffraction orders become evanescent. The only propagating intensity is due to the zeroth order, and the grating behaves as a uniaxial (or biaxial) crystal with its optical axes parallel and perpendicular to the grating stripes. The threshold period below which only the zeroth order is propagating is given by

$$\Lambda^{th} = \frac{\lambda}{n_1 \sin \zeta + (n_2^2 - n_1^2 \sin^2 \theta)^{1/2}}.$$
(2.1)

Here,  $\theta$  is the azimuth relative to the grating stripes,  $\zeta$  is the angle of incidence,  $n_1$  and  $n_2$  are the refractive indices of the grating stripes, and  $\lambda$  is the wavelength of the incident light. Sub-wavelength gratings can be either space-invariant or space-variant, and have been used for fabricating anti-reflection coatings (Grann, Moharam and Pommet [1994]), artificial refractive index distribution (Mait, Prather and Mirotznik [1999]), polarization-selective computer-generated holograms (Xu, Tyan, Sun, Fainman, Cheng and Scherer [1996]), optical filters



Fig. 1. Illustration of a one-dimensional periodic binary sub-wavelength grating. The grating period  $\Lambda$  comprises alternating stripes with refractive indexes  $n_1$  and  $n_2$  and widths  $t_1$  and  $t_2$ , respectively.  $q = t_1/\Lambda$  is defined as the duty cycle,  $\zeta$  is the incidence angle and  $\theta$  is the azimuthal angle. Note that the period is smaller than the incident wavelength,  $\lambda$ .

(Puscasu, Spencer and Boreman [2000]), wave plates (Brundrett, Glytsis and Gaylord [1996]) and polarizers (Bird and Parrish [1960]). Their use can be dated back to 1888 when Heinrich Hertz used sub-wavelength metal stripe gratings as radiowave polarizers (Hertz [1893]).

Gratings play an important role in optics. Their use has helped lay the foundations of such fields as spectroscopy and diffractive optics. Loosely speaking, when the variations of the surface relief or index modulation are slow compared to the wavelength,  $\lambda$ , the polarization of the incident wave can be neglected and approximate scalar theories can be used (Goodman [1996], Hasman, Davidson and Friesem [1991]). However, as the period of the grating decreases and becomes comparable or smaller than the wavelength, vectorial effects become more dominant, and rigorous electromagnetic theories are needed. Unfortunately, few rigorous analytical solutions are known, and the calculation of diffraction from such gratings generally requires numerical methods (Challener [1996], Guizal and Felbacq [1999]). The most commonly used method for these calculations is rigorous coupled wave analysis (RCWA), which was formulated by Moharam and Gaylord [1986]. An unfortunate drawback of RCWA is that it converges slowly for metallic lamellar gratings. This problem was addressed by Lalanne and Morris [1996] who reformulated the eigenvalues problem to achieve highly improved convergence rates, thereby extending the usefulness of RCWA-related methods. Another common method is the finite-difference time-domain method (FDTD) (see, for example, Mirotznik, Prather, Mait, Beck, Shi and Gao [2000] and Jiang and Nordin [2000]). However, despite the success of RCWA and other numerical approaches, they tend to be calculation-intensive and offer very little intuitive insight into sub-wavelength grating problems. For this reason, approximate methods are often sought.

The simplest approximate model for sub-wavelength gratings is the classical *form birefringence* (see Born and Wolf [1999]). This zero-order approximation gives the effective refractive indices of a binary sub-wavelength grating with the geometry depicted in fig. 1 as

$$n_{TE}^2 = q n_1^2 + (1 - q) n_2^2, (2.2)$$

$$n_{TM}^2 = \frac{n_1^2 n_2^2}{q n_2^2 + (1-q) n_1^2},$$
(2.3)

where the subscripts *TE* and *TM* denote light that has been polarized parallel and perpendicular to the grating stripes, respectively;  $n_1$  and  $n_2$  denote the refractive indices of the materials that comprise the grating, and  $q = t_1/\Lambda$  is the duty cycle of the grating, i.e. the relative portion of the material with refractive index  $n_1$  within the grating. Thus, the grating is replaced by an effective layer consisting

of a uniaxial crystal. If the grating period is not binary, then it is approximated with a step function, and the effective refractive indices for each step are calculated using eqs. (2.2) and (2.3). The structure is then replaced with a multilayer stack whose properties can be calculated using transfer-matrix methods (Macleod [1989], Yeh [1979]).

Some very important results regarding sub-wavelength gratings were presented by Rytov [1956]. He showed that the effective refractive indices for a subwavelength grating could be found from a pair of transcendental equations,

$$\binom{n_1^2 - n_{TE}^2}{1/2} \tan\left[\pi \frac{\lambda}{\Lambda} (1-q) \left(n_1^2 - n_{TE}^2\right)^{1/2}\right]$$
  
=  $-\left(n_2^2 - n_{TE}^2\right)^{1/2} \tan\left[\pi \frac{\lambda}{\Lambda} q \left(n_2^2 - n_{TE}^2\right)^{1/2}\right],$  (2.4)

$$\frac{(n_1^2 - n_{TM}^2)^{1/2}}{n_1^2} \tan\left[\pi \frac{\lambda}{\Lambda} (1 - q) \left(n_1^2 - n_{TM}^2\right)^{1/2}\right] \\ = \frac{-(n_2^2 - n_{TM}^2)^{1/2}}{n_2^2} \tan\left[\pi \frac{\lambda}{\Lambda} q \left(n_2^2 - n_{TM}^2\right)^{1/2}\right],$$
(2.5)

where  $\lambda$  is the incident wavelength and  $\Lambda$  is the grating period. Developing these equations into a Taylor series yields second-order approximations,

$$n_{TE}^{(2)} = \left\{ \left[ n_{TE}^{(0)} \right]^2 + \frac{1}{3} \left[ \frac{\pi \Lambda}{\lambda} q \left( 1 - q \right) \right]^2 \left( n_2^2 - n_1^2 \right)^2 \right\}^{1/2},$$
(2.6)

$$n_{TM}^{(2)} = \left\{ \left[ n_{TM}^{(0)} \right]^2 + \frac{1}{3} \left[ \frac{\pi \Lambda}{\lambda} q \left( 1 - q \right) \right]^2 \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right)^2 \left[ n_{TM}^{(0)} \right]^6 \left[ n_{TE}^{(0)} \right]^2 \right\}^{1/2},$$
(2.7)

where  $n_{TM}^{(0)}$  and  $n_{TE}^{(0)}$  are the zero-order solutions of eqs. (2.4) and (2.5) provided by eqs. (2.2) and (2.3). Further research into form birefringence was conducted by Bouchitte and Petit [1985] using homogenization techniques. They rigorously proved that any refractive index distribution can be replaced by a stratified layer as long as the period of the grating tends to zero.

The main difficulty in realizing sub-wavelength structures is their small feature size which requires the use of advanced and often creative photolithographic techniques. Roughly speaking, there are three methods for realizing these elements: indirect writing (see Bowden [1994]), direct writing (see Warren, Smith, Vawter and Wendt [1995]) and interference writing (see Enger and Case [1983]). In the indirect writing process, a mask of the element is initially made. This mask is usually a glass substrate onto which the relief pattern of the element has been

[4, § 2

placed. This is usually done by first coating the glass with a metal and then coating the metal with a photoresist. The pattern is developed onto the photoresist using either a laser or an electron beam, and then etched, leaving the desired pattern on the mask. The element can then be realized by imaging the mask onto the photoresist-coated substrate. The photoresist on the substrate is then developed and a copy of the element can be made. If the pattern is not binary, then it is necessary to make separate masks for the different layers. The main advantage of this method is that it enables relatively cheap reproduction of a single element. This technique was used by Deguzman and Nordin [2001] to fabricate a circular polarizer for the mid-infrared regime.

The second method used is direct writing. In this technique, instead of making a mask, a laser (usually UV) or electron beam is used to imprint the pattern directly onto the substrate. The substrate is first coated with photoresist, and the pattern is written directly onto this coated substrate. The pattern is then developed, and a single copy of the element is generated. An advantage of this method is that it offers high resolution, especially when electron beams are used. For this reason, it is widely used in academic research of sub-wavelength gratings for the visible region (Lopez and Craighead [1998], Warren, Smith, Vawter and Wendt [1995]). The downside is that the production time is long and the fabrication is very expensive. Therefore, this method is not widely used for commercial production.

Interference recording is the most commonly used method for the fabrication of homogeneous one-dimensional gratings (Brundrett, Gaylord and Glytsis [1998], Nordin, Meier, Deguzman and Jones [1999]). The sub-wavelength lines are produced by the interference of ultraviolet or blue laser light, leading to periods of around 200 nm. This technique is very useful in the formation of space-invariant gratings, and simple space-variant structures can be achieved by incorporating simple computer-originated phase masks into the interferometer. However, its use in forming intricate space-variant sub-wavelength gratings is rather limited.

There are also differences in the fabrication of metal and dielectric subwavelength gratings. Metal sub-wavelength gratings are usually fabricated using a lift-off process (Doumuki and Tamada [1997]). After the pattern has been transferred to the photoresist-coated substrate by either direct or indirect writing, the photoresist is developed. The substrate is then coated with metal, and the photoresist removed. In this manner, the metal remains only in the areas that were clear of photoresist after development. This technique is very useful in the realization of thin metal stripes with sub-micron features. Since the metal stripes in a subwavelength grating need not be much thicker than the skin depth, this technique is well suited for the realization of space-variant metal-stripe sub-wavelength gratings for the IR and visible spectra. On the other hand, dielectric gratings are realized using etching techniques.

Since the features of the sub-wavelength gratings are very small, and since the depth-feature size aspect ratio is usually large, it is important to choose a technique that has a large degree of anisotropy. Dry etching is usually more suitable than wet etching. In particular, reactive ion etching is especially useful (Lopez and Craighead [1998], Deguzman and Nordin [2001], and Nordin, Meier, Deguzman and Jones [1999]). After the photoresist on the substrate is developed, the element is placed in a vacuum chamber and subjected to a bombardment of a plasma mixture. The characteristics of the plasma are determined by the choice of etching technique. The areas on the substrate that were coated by photoresist remain untouched, whereas the areas that were exposed to the plasma are etched away. In this way a relief pattern is achieved on the substrate and a grating is realized.

Bomzon, Kleiner and Hasman [2001a] developed a novel method for designing and realizing nonuniformly polarized beams using computer-generated spacevariant sub-wavelength gratings. Their design is based on determining the local period and direction of the grating at each point, forming space-varying polarizers or wave plates that convert uniformly polarized light into any desired spacevariant polarization. Their gratings are continuous, thereby guaranteeing the continuity of the electromagnetic field.

# 2.1.2. Space-variant polarization-state manipulation by use of sub-wavelength metal gratings

Sub-wavelength metal stripe gratings are usually used as homogeneous spacevariant polarizers (see Glytsis and Gaylord [1992], Honkanen, Kettunen, Kuittinen, Lautanen, Turunen, Schnabel and Wyrowski [1999], Schnabel, Kley and Wyrowski [1999] or Astilean, Lalanne and Palamaru [2000]). Sometimes, however, a different polarization state is required at each location. Bomzon, Kleiner and Hasman [2001b, 2001c, 2001d] demonstrated an innovative method for designing, analyzing and realizing computer-generated space-variant metal-stripe polarization elements. This method is based on determining the local direction and period of a sub-wavelength metal-stripe grating using vectorial optics to obtain any desired continuous polarization change, hence, completely suppressing any diffraction arising from polarization discontinuity. Analysis of the element can then be performed using an original method combining RCWA and Jones calculus, in which the element is represented as a space-varying Jones matrix, which is defined by the local period and orientation of the grating. Gratings are typically defined by a grating vector that is perpendicular to the grating stripes. A space-varying grating can therefore be described by the vector

$$\mathbf{K}_{g}(x, y) = K_{0}(x, y) \cos(\theta(x, y))\hat{\mathbf{x}} + K_{0}(x, y) \sin(\theta(x, y))\hat{\mathbf{y}}, \qquad (2.8)$$

where  $K_0(x, y)$  is the spatial frequency of the grating,  $\theta$  is the direction of the vector, and  $\hat{\mathbf{x}}$ ,  $\hat{\mathbf{y}}$  are the unit vectors along the *x*-axis and the *y*-axis, respectively. In order for such a grating to be physically realizable in a continuous way,  $\mathbf{K}_g$  should be a conserving vector, i.e.,  $\nabla \times \mathbf{K}_g = 0$ , or more explicitly,

$$\frac{\partial K_0}{\partial y}\cos(\theta) - K_0\sin(\theta) \left[\frac{\partial \theta}{\partial y}\right] = \frac{\partial K_0}{\partial x}\sin(\theta) + K_0\cos(\theta) \left[\frac{\partial \theta}{\partial x}\right].$$
 (2.9)

This necessary restraint is placed on  $K_0(x, y)$  to enable a continuous grating with a local groove direction  $\theta(x, y)$  to exist. Once the grating vector is determined, the grating function  $\phi_g(x, y)$  can be found by integrating  $\mathbf{K}_g$  along any arbitrary path in the x-y plane so that  $\nabla \phi_g = \mathbf{K}_g$ . A Lee-type (Lee [1974]) binary subwavelength structure mask described by the grating function  $\phi_g(x, y)$  can be realized using high-resolution laser lithography. The amplitude transmission for such a Lee-type binary mask can be derived as

$$t(x, y) = U_s [\cos(\phi_g) - \cos(\pi q)],$$
 (2.10)

where  $U_s$  is the unit step function, defined by

$$U_{s}(\eta) = \begin{cases} 1, & \eta \ge 0, \\ 0, & \eta < 0, \end{cases}$$
(2.11)

and q is the duty cycle of the grating.

Bomzon, Kleiner and Hasman [2001b] have applied this method to the design of a space-variant polarization element, which enables the transformation of circularly polarized light into a wave with a direction of polarization that is a linear function of the *x*-coordinate. The element was fabricated as metal-stripe gratings on GaAs and ZnSe wafers using lift-off techniques. Figure 2(a) shows the magnified geometry of such a computer-generated mask with the resulting transmission axis varying in the *x*-direction from 0° to 90°. The continuity of the grating is clearly apparent. Figure 2(b) shows experimental measurements of the azimuthal angle for a circularly polarized CO<sub>2</sub> laser beam transmitted through the spacevariant polarizer. These experimental results, based on complete space-variant Stokes-parameter measurement (see Collett [1993]), revealed 98.6% overall polarization purity, taking into account the azimuthal and ellipticity deviations.



Fig. 2. (a) Magnified illustration of the computer-generated space-variant polarization element geometry. (b) Experimental measurement of the two-dimensional space-variant polarization orientations. The arrows indicate the direction of the large axis of the local polarization ellipse. (From Bomzon, Kleiner and Hasman [2001a].)

## 2.1.3. Formation of linearly polarized light with axial symmetry by using space-variant sub-wavelength dielectric gratings

Recent years have witnessed a growing interest in beams of a transversally spacevariant polarization state. One of the most interesting types of such beams is the linearly polarized axial symmetric beam (LPASB). LPASBs are characterized by their polarization orientation  $\psi(\omega) = m\omega + \psi_0$ , where *m* is the polarization order number,  $\omega$  is the azimuthal angle of the polar coordinate system, and  $\psi_0$  is the initial polarization orientation for  $\omega = 0$ . Figure 3(a) illustrates LPASBs of polarization order numbers m = 1 and m = 2. Note that LPASBs have a singularity of their polarization state at the beam axis and, therefore, have a vectorial vortexlike structure. The most renowned members of the LPASB family are the radial  $(m = 1, \psi_0 = 0)$  and azimuthal  $(m = 1, \psi_0 = \frac{1}{2}\pi)$  beams, which are extensively used for the improvement of applications such as particle acceleration, atom trap-



Fig. 3. (a) Illustration of linearly polarized light with axial symmetry with different polarization orders. (b) Geometrical definition of the grating vector. (From Niv, Biener, Kleiner and Hasman [2003].)

ping, optical tweezers (Allen, Padgett and Babiker [1999]), material processing (Niziev and Nesterov [1999]), and tight focusing (Quabis, Dorn, Eberler, Glöckl and Leuchs [2000]).

Two separate conditions have to be met if we wish to convert circularly polarized light into a LPASB using sub-wavelength gratings. The first is converting the circularly polarized light into linearly polarized light by inducing  $\frac{1}{2}\pi$  retardation on the incident wave. The second is creating the proper local polarization direction. The first condition is met by choosing the correct shape for the subwavelength grooves while the second is fulfilled by creating local groove orientation of the form

$$\theta(\omega) = \psi(\omega) - \frac{1}{4}\pi = m\omega + \psi_0 - \frac{1}{4}\pi.$$
(2.12)

An axially symmetric space-variant sub-wavelength grating is typically described by a grating vector of the form

$$\mathbf{K}_{g}(r,\omega) = K_{0}(r,\omega) \Big[ \cos(\theta(r,\omega) - \omega) \hat{r} + \sin(\theta(r,\omega) - \omega) \hat{\omega} \Big], \qquad (2.13)$$

where  $\hat{r}$ ,  $\hat{\omega}$  are unit vectors in polar coordinates (fig. 3(b)), and  $K_0(r) = 2\pi/\Lambda(r,\omega)$  is the local spatial frequency for a grating of local period  $\Lambda(r,\omega)$ . Next, to ensure the continuity of the grating, we require that  $\nabla \times \mathbf{K}_g = 0$ , resulting in a differential equation that can be solved to yield the local grating period. The solution to this problem yields  $K_0(r) = (2\pi/\Lambda_0)(r_0/r)^m$ , where  $\Lambda_0$  is the local sub-wavelength period at  $r = r_0$ . Integrating  $\mathbf{K}_g$  over an arbitrary path yields the desired grating function (defined such that  $\nabla \phi_g = \mathbf{K}_g$ ) as

$$\begin{split} \phi_g(r,\omega) &= 2\pi r_0 (r_0/r)^{m-1} \sin\left[(m-1)\omega + \psi_0 - \frac{3}{4}\pi\right] / \left[(m-1)\Lambda_0\right] \\ \text{for } m \neq 1, \\ \phi_g(r,\omega) &= (2\pi r_0/\Lambda_0) \left[\ln(r/r_0)\cos\left(\psi_0 - \frac{1}{4}\pi\right) + \omega\sin\left(\psi_0 - \frac{1}{4}\pi\right)\right] \\ \text{for } m = 1. \end{split}$$
(2.14b)

Niv, Biener, Kleiner and Hasman [2003] realized Lee-type binary grating functions for  $m = \frac{1}{2}$ , 1,  $\frac{3}{2}$ , 2 and  $\psi_0 = \frac{1}{4}\pi$ . The gratings were fabricated on 500 µmthick GaAs wafers for CO<sub>2</sub> laser radiation with a wavelength of 10.6 µm, with  $\Lambda_0 = 2$  µm,  $r_0 = 4.7$  mm, and a maximum radius of 6 mm. They formed the gratings with a maximum local period of 3.2 µm in order not to exceed the Wood anomaly of GaAs.

Figure 4(a) shows the intricate geometry of a sub-wavelength stripe grating designed to convert circularly polarized light into LPASB. Figure 4(b) shows an image obtained by using a linear polarizer as an analyzer. Note that for each beam the polarization state repeats itself 2m times. Experimental values of the local



Fig. 4. (a) Magnified geometry of the sub-wavelength gratings for different polarization orders,  $m = \frac{1}{2}$ , 1,  $\frac{3}{2}$ , 2. (b) Experimental intensity distributions, directly after the gratings of different polarization orders, of the beams emerging from a linear polarizer–analyzer. (c) Measured local azimuthal angles of the beams. (From Niv, Biener, Kleiner and Hasman [2003].)

azimuthal angle  $\psi$  at each point are shown in fig. 4(c). The manipulation resulted in a high polarization purity of over 98% in the desired direction.

Additional insight can be obtained by performing polarization-state and phase analysis of the resulting beam. By representing the element as a space-variant Jones matrix, the resultant wavefront can be found for any incident polarization (see, for example, Hasman, Bomzon, Niv, Biener and Kleiner [2002]). For a space-varying quarter-wave plate and incident right-hand circular polarization, the Jones vector of the resultant beam is

$$\mathbf{E}_{\text{out}}(r,\omega) = \begin{pmatrix} \cos(m\omega + \psi_0)\\ \sin(m\omega + \psi_0) \end{pmatrix} \exp\left[-i(m\omega + \psi_0)\right].$$
(2.15)

Using the rule proposed by Pancharatnam [1956] for comparing the phases of two light beams in different states of polarization, we can now calculate the space-variant Pancharatnam phase of the transmitted beam as

$$\varphi_p = \arg \langle \mathbf{E}(r, \omega), \mathbf{E}(R, 0) \rangle, \tag{2.16}$$

where  $\arg \langle \mathbf{E}(r, \omega), \mathbf{E}(R, 0) \rangle$  is the argument of the inner product of the two vectors and (R, 0) are the coordinates of the point on the resultant beam with respect to which the phase is measured. This calculation yields

$$\varphi_p = \arg[\cos(m\omega + \psi_0)] - m\omega. \tag{2.17}$$

This phase modification results solely from the polarization manipulation and is purely geometrical in nature (Bomzon, Niv, Biener, Kleiner and Hasman [2002b]). The beam displays a Pancharatnam phase ramp with a helical structure similar to those found in scalar optical vortices; therefore, we define the topological Pancharatnam charge of the beam as

$$l_P = (1/2\pi) \oint \nabla \varphi_p \,\mathrm{d}s = -m, \qquad (2.18)$$

where ds is an infinitesimal distance in the direction of the integration path. Note that in our case the Pancharatnam charge and the polarization order number are equal in magnitude and opposite in sign. This charge can be modified by transmission of the beam through a spiral phase element of the form  $\exp(il_d\omega)$ , ( $l_d$  integer), whereby a topological charge of  $l_d$  is added to the beam.

Figures 5(a) and 5(c) show the calculated real parts of the instantaneous fields of LPASBs. Figure 5(a) shows the fields of the beams formed by use of the gratings only, for  $m = \frac{1}{2}$ , 1,  $\frac{3}{2}$ , 2, and  $\psi_0 = \frac{1}{4}\pi$ . Figure 5(c), on the other hand, shows the beams that are created when, in addition to the grating, the waves of fig. 5(a) are also transmitted through spiral phase elements bearing topological charge  $l_d = m$ . The result is the cancellation of the Pancharatnam phase while



Fig. 5. Calculated real part of the instantaneous vector fields for beams (a) emerging from the gratings only, and (c) with additional spiral phase element of  $l_d = m$ . (b, d) Experimental far-field images for the beams and their calculated (solid curves) and measured (solid circles) cross-sections. [(b) and (d) correspond to (a) and (c), respectively.] (From Niv, Biener, Kleiner and Hasman [2003].)

maintaining both the same space-variant polarization directions as well as the same polarization order number *m*. In cases where the polarization order number is a half-integer (e.g.,  $m = \frac{1}{2}, \frac{3}{2}$ ), the fields of the beams having Pancharatnam phase (fig. 5(a)) are continuous, whereas the fields of fig. 5(c) without the Pancharatnam phase are discontinuous. This indicates that the formation of continuous LPASBs with a half-integer polarization order is possible only for beams having a topological Pancharatnam charge.

Figure 5 also shows the experimental far-field images of these fields, with figs. 5(b) and 5(d) corresponding to figs. 5(a) and 5(c), respectively, as well as the calculated and measured cross-sections. Note that the beams emerging from only the gratings, fig. 5(b), exhibit far-field images with bright centers, while the beams undergoing a cancellation of the Pancharatnam phase exhibit distinct far-field images with dark centers, fig. 5(d). There is a close connection between the instantaneous electric field (figs. 5(a), 5(c)) and the appearance of a bright or dark spot at the far-field image of the beams. When the integral of the real part of the instantaneous electric field around the beam axis is zero,

$$\operatorname{Re}\left(\int_{0}^{2\pi} \mathbf{E}(\omega) \,\mathrm{d}\omega\right) = \begin{pmatrix} 0\\0 \end{pmatrix},\tag{2.19}$$

a dark spot at the far-field is obtained; conversely, a nonzero sum reveals a bright spot. The experimental results indicate that LPASBs with identical polarization orders, but of different Pancharatnam phases, propagate in different ways, which emphasizes the relevance of correct phase determination in the propagation of space-variant polarization beams.

Another point of interest is the angular momentum of such beams. For a scalar wave, the angular momentum in the direction of propagation per unit energy is given by  $j_z = (l + \sigma)/(2\pi\nu)$  (see, for example, Allen, Padgett and Babiker [1999]), where *l* is the topological charge,  $\sigma$  is the helicity (±1 for circular polarization) and  $\nu$  is the optical frequency of the beam. Using this rule and the decomposition of  $\mathbf{E}_{out}$  into circular polarization states yields the angular momentum of LPASBs as

$$j_z = \frac{1}{2} \sum_{i=L,R} (l_i + \sigma_i) / (2\pi\nu) = (l_d - m) / (2\pi\nu) = l_p / (2\pi\nu),$$

where L and R indicate the components with left and right circular polarization, respectively, and show that the angular momentum of these waves is given by the topological Pancharatnam charge. This result introduces a connection between angular momentum and topological Pancharatnam charge.

The formation of both radial and azimuthal beams can also be achieved by the use of methods such as interferometric techniques, intracavity summation of two

orthogonally polarized  $\text{TEM}_{01}$  modes, or by using liquid-crystal devices, as will be discussed in the following subsections.

### 2.2. Space-variant vectorial fields obtained by using interference methods

In this subsection we present laser resonator configurations and interferometric techniques in which the polarization in different parts of the output beam can be varied, generating, in effect, an output beam with space-variant polarization. Two interferometric techniques for converting a linearly polarized laser beam into a radially polarized beam with uniform azimuthal intensity were presented by Tidwell, Ford and Kimura [1990]. In the first method, linearly polarized beams with intensity profiles tailored using a modified laser or an apodization filter are combined in separate experiments to produce radially polarized light. The linear polarization-combining technique uses a Mach–Zehnder arranged as a 90° rotation shear interferometer operating on the null fringe, as shown in fig. 6(a). The second technique, shown in fig. 6(b), uses circularly polarized light instead of linearly polarized light, and a unique spiral phase-delay plate to produce the required phase profile. In a later study, Tidwell, Kim and Kimura [1993] presented a hybrid of these two earlier approaches for the conversion of a linearly polarized  $CO_2$ laser beam into a radially polarized beam. The result is a double-interferometer system that is able to convert any linearly polarized laser beam profile into a radi-



Fig. 6. Mach–Zehnder interferometer configurations used to produce radially polarized beams. (a) 90° rotational shear interferometer that converts a sinusoidally varying linearly polarized beam. (b) Conventional Mach–Zehnder for converting a general linearly polarized beam using spiral delay of circularly polarized light. (M = mirror, BS = beam splitter, P = polarizer, PS = periscope, SPDP = spiral phase delay plate.) (From Tidwell, Ford and Kimura [1990].)

4, § 2]

ally polarized one with high efficiency. Using this method, the authors were able to generate a beam that was  $\sim$  92% radially polarized and contained  $\sim$  85% of the input power.

Azimuthal and radial polarizations have also been obtained by inserting polarization-selective elements into a laser resonator. Pohl [1972] inserted a birefringent calcite crystal with the principal axis along the z-axis (z-cut), into a pulsed ruby laser in order to discriminate between azimuthal and radial polarization. Wynne [1974] generalized this method and showed experimentally, with a wavelength-tunable dye laser, that it is possible to select either the azimuthally or the radially polarized modes. Mushiake, Matsumura and Nakajima [1972] used a conical intra-cavity element to select a radially polarized mode. The conical element introduced low reflection losses to the radially polarized mode but high reflection losses to the azimuthally polarized mode. This method is somewhat similar to applying a Brewster window to obtain linear polarization. Similarly, Tovar [1998] suggested using complex Brewster-like windows, of either conical or helical shape, to select radially or azimuthally polarized modes. Nesterov, Niziev and Yakunin [1999] replaced one of the mirrors of a high-power CO<sub>2</sub> laser with a sub-wavelength diffractive element. This element consisted of either concentric circles (for selecting azimuthal polarization) or straight lines through a central spot (for selecting radial polarization) to obtain different reflectivities for the azimuthal and radial polarizations. Experimentally, a high output power of 1.8 kW was obtained, but the polarization purity was relatively low, with mixed transverse mode operation. Liu, Gu and Yang [1999] analyzed a resonator configuration into which two sub-wavelength diffractive elements were incorporated, to obtain a different fundamental mode pattern for two different polarizations. Oron, Blit, Davidson, Friesem, Bomzon and Hasman [2000] presented a method for efficiently obtaining essentially pure azimuthally and radially polarized beams directly from a laser. The method is based on the selection and coherent summation of two linearly polarized transverse modes that exist inside the laser resonator; specifically, two orthogonally polarized  $TEM_{01}$  modes. Figure 7(a) depicts an azimuthally polarized beam, obtained by coherent summation of a y-polarized  $\text{TEM}_{01(x)}$  mode and an x-polarized  $\text{TEM}_{01(y)}$  mode. Figure 7(b) shows a radially polarized beam, obtained by the coherent summation of an x-polarized  $\text{TEM}_{01(x)}$  mode and a y-polarized  $\text{TEM}_{01(y)}$  mode. The modes are selected by inserting phase elements that permit significant mode discrimination into the laser resonator when properly combined. The laser resonator configuration in which specific transverse modes are selected and coherently summed is schematically shown in fig. 8.



Fig. 7. Coherent superposition of two orthogonally polarized TEM<sub>01</sub> modes to form azimuthally and radially polarized beams. (a) Azimuthally (θ) polarized doughnut beam. (b) Radially (r) polarized doughnut beam. (From Oron, Blit, Davidson, Friesem, Bomzon and Hasman [2000].)



Fig. 8. Laser resonator configuration with a discontinuous phase element (DPE) for forming an azimuthally or radially polarized beam. (From Oron, Blit, Davidson, Friesem, Bomzon and Hasman [2000].)

#### 2.3. Space-variant vectorial fields obtained by using liquid-crystal devices

Liquid crystals have been selected as optical materials because of the flexibility they provide in designing new optical components. Stalder and Schadt [1996] suggested the use of a liquid-crystal device for generating linearly polarized light with axial symmetry using two optical effects. In the first case they realigned the incoming linearly polarized light by using the twisted nematic effect. The study made use of a basic cell consisting of one unidirectionally and one circularly rubbed alignment layers and filled with nematic liquid-crystal. The local liquid-



Fig. 9. Generation of radially and azimuthally polarized light. (From Stalder and Schadt [1996].)

crystal orientation in the cell is that of a twisted cell with a variable twist angle defined by the local alignment layers. The generation of radially and azimuthally polarized light using this type of cell is illustrated in fig. 9. The authors also proposed the generation of light having a polarization order number m = 2 by using the effect of  $\frac{1}{2}\lambda$  wave plates in liquid-crystal devices that exhibit spatially variable alignment layers. In this study, the liquid-crystal cell consists of two circularly rubbed alignment layers with aligned centers of symmetry. The substrate glasses are coated with transparent electrodes. By applying the correct electric field between the electrodes, the cell becomes a  $\frac{1}{2}\lambda$  plate whose fast axis rotates one full rotation around the beam axis, for a given  $\lambda$ . Two-dimensional encoding of the polarization state of a laser beam was demonstrated by Davis, McNamara, Cottrell and Sonehara [2000] using a parallel-aligned liquid-crystal spatial light modulator (LCSLM). Each pixel of the LCSLM acts as a voltage-controlled wave plate that is capable of phase modulation over  $2\pi$  rad at an argon laser wavelength of 514.5 nm.

A liquid-crystal-based polarization grating was reported by Wen, Petschek and Rosenblatt [2002]. In this case, the orientation of one or both of the polarization eigenvectors is altered as light passes through the liquid-crystal cell. In one of its simplest forms, the grating permits only odd-order diffraction peaks. The researchers also developed more complex gratings, including a grating that rotates both polarization components in tandem, while simultaneously applying relative phase retardation. For an appropriate rotation and retardation, the device simulates a blazed grating for circularly polarized light. In addition, because the polar orientation of the liquid-crystal director can be controlled by an electric field applied across semitransparent indium tin oxide electrodes, one can switch the cell from grating mode to straight-throughput mode.

A method that permits the accurate generation of arbitrary complex vector wave fields by using a binary optical element was described by Neil, Massoumian, Juškaitis and Wilson [2002]. The binary phase modulation was achieved by a reconfigurable ferroelectric liquid-crystal spatial light modulator (FLCSLM). However, the binarization process leads to a maximum efficiency in the first diffracted order of just 40.5%, and the use of only half of the light in the first diffracted order for each polarization reduces this to 20.25%. In addition, the form of particular desired complex fields (the amplitude and phase of the components) could lead to additional reductions in efficiency. Finally, Davis, Adachi, Fernández-Pousa and Moreno [2001] used a parallel-aligned active matrix nematic liquid-crystal spatial light modulator to realize a spatially variable wave plate for which the relative phase retardation varies linearly along the *x*-direction as  $\phi = 2\pi x/\Lambda$ , where  $\Lambda$  is the period. Interestingly, the diffracted order is linearly polarized in this case, regardless of the incoming polarization state. This is contrary to the case presented in Section 3 and in the work of Hasman, Bomzon, Niv, Biener and Kleiner [2002] in which constant retardation and space-variant sub-wavelength gratings produce diffracted orders that are always circularly polarized (see Cincotti [2003]).

### 2.4. Alternative methods

Another technique to achieve space-variant polarization-state manipulation is by holographic recording using polarization-sensitive materials. In this procedure, a space-variant polarized beam illuminates a polarization-sensitive recording material. The beam is typically generated interferometrically, as described in Section 2.2. When the hologram is illuminated using only one arm of the interferometer, a full reconstruction of the space-variant polarized beam emerges. This method was first described by Kakichashvili [1972], who used the Weigert effect (optical anisotropy induced in a fine-grain silver chloride photographic emulsion by exposure to a beam of linearly polarized light) for holographic recording and reconstruction of the polarization state of a beam. However, further progress in polarization holography was hampered mainly by the low efficiency of the recorded holograms (<1%). This situation remained until Todorov, Nikolova and Tomova [1984] introduced a new polarization-sensitive material: a methyl orange azo dye in a polyvinyl alcohol matrix. They demonstrated that high-efficiency polarization holograms (>35%) can be recorded repeatedly. This seminal work triggered a variety of research aiming to find ever more efficient and stable materials. Among these researchers, Ciuchi, Mazzulla and Cipparrone [2002] demonstrated long-time stability (over a year's period) in two kinds of azo-dye elastomer films, while Holme, Ramanujam and Hvilsted [1996] reported 10,000 rapid write, read and erase cycles in an azobenzene sidechain liquid-crystalline polyester. Also, a veritable wave of applications emerged mainly for the optical storage of data. Among them was that of Kawano, Ishii, Minabe, Niitsu, Nishikata and Baba [1999] which proposed digital holographic storage with polarization multiplexing by use of polyester-containing cyanoazobenzene units in the side chain. In another research Ferrari, Garbusi and Frins [2004] demonstrated phase-shifting interferometry using bacteriorhodopsin film.

It is worth mentioning that the ability to perform space-variant polarizationstate manipulations led to the development of several designing techniques. Tervo, Kettunen, Honkanen and Turunen [2003] proposed iterative design algorithms of diffractive elements for paraxial vector fields. Specifically, they showed that utilizing the local polarization state of a beam as an additional design freedom leads to more light-efficient design with minimal trade-off between diffraction efficiency and signal quality. Cincotti [2003] showed that the polarization state of the diffracted waves (higher-order waves) does not depend on the polarization state of the incoming wave but that they are fully determined by the polarization grating. She used this approach to propose a general model for polarization gratings that can be exploited for the design of such elements.

#### § 3. Geometrical phase in space-variant polarization-state manipulation

The Pancharatnam–Berry phase is a geometrical phase associated with the polarization of light. When the polarization of a beam traverses a closed loop on the Poincaré sphere, the final state differs from the initial state by a phase factor equal to half the Area ( $\Omega$ ) encompassed by the loop on the sphere (see Pancharatnam [1956], Berry [1984], and Shapere and Wilczek [1989]). In a typical experiment, the polarization of a uniformly polarized beam is altered by a series of spaceinvariant (transversely homogeneous) wave plates and polarizers, and the phase, which evolves in the time-domain, is measured by means of interference (Simon, Kimble and Sudarshan [1988], Kwiat and Chiao [1991]).

Niv, Biener, Kleiner and Hasman [2003] considered a Pancharatnam–Berry phase in the space domain. Using space-variant dielectric sub-wavelength gratings, they demonstrated conversion of circularly polarized into linearly polarized axially symmetric beams. They showed that the conversion was accompanied by a space-variant phase modification of geometrical origin that affected the propagation of the beams. An earlier study by Bomzon, Kleiner and Hasman [2001d] demonstrated a Pancharatnam–Berry phase in space-variant polarization-state manipulation using space-variant metal-stripe sub-wavelength gratings.

Frins, Ferrari, Dubra and Perciante [2000] described a method for generating arbitrary axial phase discontinuities that is based on Pancharatnam's theorem.



Fig. 10. Setup for the generation of phase dislocations.  $Q_1$  and  $Q_2$  are rectangular QWPs with their respective fast axes (F.A.) orthogonal to each other;  $P_1$  is a polarizer at 45° with respect to the axes of the QWPs;  $P_2$  is a rotatable polarizer (analyzer);  $\nu$  is the angle between the directions of the transmission axes of  $P_1$  and  $P_2$ . (From Frins, Ferrari, Dubra and Perciante [2000].)

They utilized this method for converting a bright, nondiffracting beam into a dark one. The basic concept for producing the phase dislocation is shown in fig. 10. The design consists of two quarter-wave plates (QWPs) placed side by side with their fast axes perpendicular to each other, sandwiched by two polarizers. The transmission axis of the first polarizer is at 45° with respect to the fast axes of the QWPs. Baba, Murakami and Ishigaki [2001] proposed using a space-variant geometrical phase for applications such as null interferometry. Previously, Bhandari [1997] suggested using a discontinuous spatially varying wave plate as a lens based on similar geometrical phase effects.

Zhan and Leger [2002] reported an interferometric measurement of the geometric phase in space-variant polarization manipulation. They experimentally verified it using a dichroic radial polarizer. A dichroic radial polarizer converts a circularly polarized beam into an azimuthally polarized beam with a spiral geometrical phase. A Mach–Zehnder interferometer, which is shown in fig. 11, is used to measure this spiral phase.

Conversion of an input polarization state into a space-variant polarization state has been investigated using periodic polarization gratings. Gori [1999] proposed using spatially rotating polarizers as a polarization grating while Fernández-Pousa, Moreno, Davis and Adachi [2001] proposed using a polarization grating with space-variant retardation realized by a liquid-crystal, spatial light modulator. Tervo and Turunen [2000] proposed using a polarization grating formed by spatially rotating wave plates. These authors showed that the polarization of the



Fig. 11. Mach–Zehnder interferometer setup for the spiral geometric phase measurement. (From Zhan and Leger [2002].)

diffracted orders could differ from the polarization of the incident beams. The formation of complex vectorial fields by use of polarization gratings has been discussed in greater detail in Section 2.

In the present section we consider optical phase elements based on the spacedomain Pancharatnam–Berry phase. Unlike with diffractive and refractive elements, the phase is not introduced through optical path differences, but results from the geometrical phase that accompanies space-variant polarization manipulation. Optical elements that use this effect to form a desired phase front are called Pancharatnam–Berry-phase optical elements (PBOEs). These elements are polarization dependent, thereby permitting the construction of multi-purpose optical elements that are suitable for applications such as optical switching, optical interconnects and beam splitting (see Hasman, Bomzon, Niv, Biener and Kleiner [2002]). Such elements can be realized using computer-generated spacevariant sub-wavelength dielectric gratings. Biener, Niv, Kleiner and Hasman [2002] and Hasman, Kleiner, Biener and Niv [2003] experimentally demonstrated Pancharatnam–Berry-phase diffraction gratings for  $CO_2$  laser radiation at a wavelength of 10.6  $\mu$ m, showing an ability to form complex polarization-dependent phase elements.

Figure 12 illustrates the concept of PBOEs on the Poincaré sphere. Circularly polarized light is incident on a wave plate with constant retardation and a continuously space-varying fast axis whose orientation is denoted by  $\theta(x, y)$ . Bomzon, Biener, Kleiner and Hasman [2002c] showed that since the wave plate is space varying, the beam at different points traverses different paths on the Poincaré sphere, resulting in a space-variant phase-front modification originating from the



Fig. 12. Illustration of the principle of PBOEs by use of the Poincaré sphere. (From Bomzon, Biener, Kleiner and Hasman [2002c].)

Pancharatnam–Berry phase. Their goal was to utilize this space-variant geometrical phase to form novel optical elements.

It is convenient to describe PBOEs using Jones calculus. In this representation, a wave plate with a fast axis oriented along the *y*-axis can be described by the Jones matrix

$$\mathbf{J} = \begin{pmatrix} t_x & 0\\ 0 & t_y \mathrm{e}^{\mathrm{i}\phi} \end{pmatrix},\tag{3.1}$$

where  $t_x$  and  $t_y$  are the real amplitude transmission coefficients for light which has been polarized perpendicular and parallel to the optical axes, and  $\phi$  is the retardation of the wave plate. A PBOE which contains wave plates with spacevariant orientation can be described by the space-dependent matrix

$$\mathbf{T}_{C}(x, y) = \mathbf{J}_{R}(\theta(x, y)) \mathbf{J} \mathbf{J}_{R}^{-1}(\theta(x, y)),$$
(3.2)

where  $\theta(x, y)$  is the local orientation of the optical axis and  $\mathbf{J}_R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$  is a two-dimensional rotation matrix.

For convenience, we adopt the Dirac bra-ket notation, and convert  $\mathbf{T}_C(x, y)$  to the helicity base in which  $|\mathbf{R}\rangle = (1 \ 0)^T$  and  $|\mathbf{L}\rangle = (0 \ 1)^T$  are the twodimensional unit vectors for right-hand and left-hand circularly polarized light, and T denotes transposition. In this base, the space-variant polarization operator is described by the matrix  $\mathbf{T}(x, y) = \mathbf{U}\mathbf{T}_C\mathbf{U}^{-1}$ , where  $\mathbf{U} = 2^{-1/2} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$  is a unitary conversion matrix. Explicit calculation of  $\mathbf{T}(x, y)$  yields

$$\mathbf{T}(x, y) = \frac{1}{2} \begin{pmatrix} t_x + t_y e^{\mathbf{i}\phi} \end{pmatrix} \begin{pmatrix} 1 & 0\\ 0 & 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} t_x - t_y e^{\mathbf{i}\phi} \end{pmatrix} \begin{pmatrix} 0 & \exp[\mathbf{i}2\theta(x, y)]\\ \exp[-\mathbf{i}2\theta(x, y)] & 0 \end{pmatrix}.$$
 (3.3)

242



Fig. 13. Operation of polarization diffraction gratings. A beam with polarization  $|\mathbf{E}_{in}\rangle$  is incident on the polarization grating. The resulting beam comprises three polarization orders: the  $|\mathbf{E}_{in}\rangle$  polarization order, which maintains the original polarization and does not undergo phase modification; the  $|\mathbf{R}\rangle$ polarization order that is right-hand circularly polarized, and whose phase is modified by  $2\theta(x, y)$ ; and the  $|\mathbf{L}\rangle$  polarization order that is left-hand circularly polarized, and whose phase is modified by  $-2\theta(x, y)$ . When  $\theta(x, y)$  is periodic, the  $|\mathbf{R}\rangle$  polarization order and the  $|\mathbf{L}\rangle$  polarization order undergo diffraction, resulting in the appearance of discrete diffraction orders. (From Hasman, Bomzon, Niv, Biener and Kleiner [2002].)

Thus, for an incident plane wave with arbitrary polarization  $|\mathbf{E}_{in}\rangle$  the resulting field is

$$|\mathbf{E}_{\text{out}}\rangle = \eta_E |\mathbf{E}_{\text{in}}\rangle + \eta_R e^{i2\theta(x,y)} |\mathbf{R}\rangle + \eta_L e^{-i2\theta(x,y)} |\mathbf{L}\rangle, \qquad (3.4)$$

where  $\eta_E = \frac{1}{2}(t_x + t_y e^{i\phi}), \eta_R = \frac{1}{2}(t_x - t_y e^{i\phi}) \langle \mathbf{E}_{in} | \mathbf{L} \rangle$  and  $\eta_L = \frac{1}{2}(t_x - t_y e^{i\phi}) \times$  $\langle \mathbf{E}_{in} | \mathbf{R} \rangle$  are the complex field efficiencies and  $\langle \alpha | \beta \rangle$  denotes the inner product. Figure 13 is a graphic representation of the results of eq. (3.4). It shows that  $|\mathbf{E}_{out}\rangle$ comprises three polarization orders: the  $|\mathbf{E}_{in}\rangle$  polarization order, the  $|\mathbf{R}\rangle$  polarization order and the  $|\mathbf{L}\rangle$  polarization order. The  $|\mathbf{E}_{in}\rangle$  polarization order maintains the polarization and phase of the incident beam, whereas the phase of the  $|\mathbf{R}\rangle$  polarization order is equal to  $2\theta(x, y)$ , and the phase of the  $|\mathbf{L}\rangle$  polarization order is equal to  $-2\theta(x, y)$ . We note that the phase modification of the  $|\mathbf{R}\rangle$  and  $|\mathbf{L}\rangle$ polarization orders results solely from local changes in polarization and is therefore geometrical in nature. Using eq. (3.4) we can calculate the Pancharatnam phase front of the resulting wave. Pancharatnam's definition for the phase between two beams of different polarization is  $\varphi_p(x, y) = \arg[\langle \mathbf{E}_{out}(0, 0) | \mathbf{E}_{out}(x, y) \rangle].$ For incident  $|\mathbf{R}\rangle$  polarization,  $\varphi_p(x, y) = -\theta + \arctan[\cos\phi \tan\theta] = -\theta +$  $\arctan[\sin(2\chi)\tan\theta]$ , where  $\chi$  is the ellipticity of the resulting beam. Geometrical calculations show that  $\varphi_p$  is equal to one half the area of the geodesic triangle,  $\Omega$ , on the Poincaré sphere defined by the pole  $|\mathbf{R}\rangle$ ,  $|\mathbf{E}_{out}(0)\rangle$ , and  $|\mathbf{E}_{out}(\theta)\rangle$  (as illustrated in fig. 12), and yields the expected Pancharatnam–Berry phase. Similar results can be obtained for any incident polarization.

A case of special interest is  $\phi = \pi$  and  $t_x = t_y = 1$ , for which we find that the diffraction efficiency is 100%, and that  $|\mathbf{R}\rangle$  polarization is completely converted into  $|\mathbf{L}\rangle$  polarization. However, despite the fact that the resulting polarization is space-invariant, the Pancharatnam phase,  $\varphi_p = -2\theta(x, y)$ , is equal to the desired geometrical phase,  $\varphi_d$ . This phase corresponds to one half of the area encompassed by two geodesic paths between the poles that form an angle of  $2\theta$  with respect to one another, as illustrated in fig. 12. This proves that the phase added to the incident beam is geometrical in nature. Note that PBOEs operate in different ways on the two helical polarizations.

To conclude, unlike conventional elements, PBOEs are not based on optical path differences, but on geometrical phase modification resulting from space-variant polarization manipulation. In Section 3.1 we will describe the design procedure for a continuous PBOE along with an example of a blazed grating, and demonstrate the ability to form more complex phase fronts such as helical beams. In Section 3.2 we will describe the quantized PBOE (QPBOE) using a different design method. In Section 3.2 we will also demonstrate the ability to form a polarization-dependent lens using a QPBOE, and will analyze propagation invariant beams formed by QPBOEs. In Section 3.3 we will demonstrate an interesting phenomenon – vectorial Talbot beams and vectorial nondiffracting beams generated by using a PBOE.

#### 3.1. Continuous Pancharatnam–Berry-phase optical elements

PBOEs can be realized by using space-variant sub-wavelength gratings. When the period of the grating is much smaller than the incident wavelength, the grating acts as a uniaxial crystal (see Section 2). Therefore, by correctly controlling the depth, structure and orientation of the grating, the desired PBOE can be made. To design a PBOE, we need to ensure that the direction of the grating stripes,  $\theta(x, y)$ , is equal to half of the desired geometrical phase, which we denote as  $\varphi_d(x, y)$ . Next we define a grating vector  $\mathbf{K}_g = K_0(x, y) [\cos(\varphi_d(x, y)/2)\hat{\mathbf{x}} + \sin(\varphi_d(x, y)/2)\hat{\mathbf{y}}]$ , where  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  are unit vectors in the *x* and *y* directions,  $K_0 = 2\pi/\Lambda(x, y)$  is the spatial frequency of the grating ( $\Lambda$  is the local sub-wavelength period) and  $\frac{1}{2}\varphi_d(x, y)$  is the space-variant direction of the vector defined so that it is perpendicular to the grating stripes at each point. Next, to ensure the continuity of the grating, thereby ensuring the continuity of the resulting optical field, we require  $\nabla \times \mathbf{K}_g = 0$ , resulting in a differential equation that can be solved to yield the local



Fig. 14. Geometry of the space-variant sub-wavelength grating as well as the geometrical phases for incident  $|\mathbf{R}\rangle$  and  $|\mathbf{L}\rangle$  polarizations. (From Bomzon, Biener, Kleiner and Hasman [2002c].)

grating period. The grating function  $\phi_g$  (defined so that  $\nabla \phi_g = \mathbf{K}_g$ ) is then found by integrating  $\mathbf{K}_g$  over an arbitrary path (see, for example, Bomzon, Kleiner and Hasman [2001a]). The realization of the grating function can be done by a Leetype binary mask. The formation of a Lee-type binary mask was explained in Section 2.1.2.

An interesting example is introduced in the work of Bomzon, Biener, Kleiner and Hasman [2002c]. They designed a PBOE that acts as a diffraction grating by requiring that  $\varphi_d = (2\pi/d)x|_{\text{mod } 2\pi}$ , where *d* is the period of the structure. They realized a Lee-type binary grating describing the grating function,  $\phi_g$ . The grating was fabricated for a CO<sub>2</sub> laser radiation with a wavelength of 10.6 µm. Two types of gratings were formed on a GaAs wafer to yield retardations of  $\phi = \frac{1}{2}\pi$  and  $\phi = \pi$ . Figure 14 illustrates the geometry of the grating, as well as the geometrical phase for incident  $|\mathbf{L}\rangle$  and  $|\mathbf{R}\rangle$  polarization states as calculated by eq. (3.4). The geometrical phases resemble blazed gratings with opposite blazed directions for incident  $|\mathbf{L}\rangle$  and  $|\mathbf{R}\rangle$  polarization states, as expected from our previous discussions.

Following their fabrication, the PBOEs were illuminated with circular and linear polarizations. Figure 15 shows the experimental images of the diffracted fields for the resulting beams, as well as their cross-sections for retardations of  $\phi = \frac{1}{2}\pi$ and  $\phi = \pi$ , respectively. When the incident polarization is circular, and  $\phi = \frac{1}{2}\pi$ , close to 50% of the light is diffracted according to the geometrical phase added to the  $|\mathbf{L}\rangle$  or  $|\mathbf{R}\rangle$  polarization orders (the direction of diffracted in the  $|\mathbf{E}_{in}\rangle$  polarization order as expected from eq. (3.4). Furthermore, the polarization of the diffracted order has switched helicity as expected. For  $\phi = \pi$ , no energy appears in the  $|\mathbf{E}_{in}\rangle$  polarization order, and the diffraction efficiency is close to 100%.



Fig. 15. Measurements of the transmitted far-field for the sub-wavelength PBOE grating for retardations of  $\phi = \frac{1}{2}\pi$  and  $\phi = \pi$  respectively, for incident (a) circular left, (b) circular right and (c) linear polarizations. (From Bomzon, Biener, Kleiner and Hasman [2002c].)

When the incident polarization is linear,  $|\mathbf{E}_{in}\rangle = 2^{-1/2}(|\mathbf{R}\rangle + |\mathbf{L}\rangle)$ , the two helical components of the beam are subject to different geometrical phases of opposite sign, and are diffracted to the  $|\mathbf{R}\rangle$  and  $|\mathbf{L}\rangle$  polarization orders in different directions. When  $\phi = \frac{1}{2}\pi$ , the  $|\mathbf{E}_{in}\rangle$  polarization order maintains the original polarization, in agreement with eq. (3.4), whereas for retardation  $\pi$  the diffraction is 100% efficient for both circular polarizations, and no energy is observable in the  $|\mathbf{E}_{in}\rangle$  polarization order.

The elements proposed here can be utilized for polarization-sensitive beam splitting and optical switches. In Section 3.1.1 we will demonstrate the ability to form helical wavefronts using continuous PBOEs. The example provided will demonstrate the ability to form a complex wavefront by using the design procedure for continuous PBOEs described in here.

## 3.1.1. Formation of helical beams by Pancharatnam–Berry-phase optical elements

Recent years have witnessed a growing interest in helical beams and their use in a variety of applications. These include trapping of atoms and macroscopic particles (see, for example, Paterson, MacDonald, Arlt, Sibbett, Bryant and Dholakia [2001] and Allen, Padgett and Babiker [1999]), transfer of orbital angular momentum to macroscopic objects (Mair, Vaziri, Weihs and Zeilinger [2001]), rotational frequency shifting, the study of optical vortices (Sacks, Rozas and Swartzlander [1998]), and specialized alignment schemes. Beams with helical (or spiral) wave fronts are described by complex amplitudes  $u(r, \omega) \propto \exp(-il\omega)$ , where r and  $\omega$ are the cylindrical coordinates - the radial coordinate and azimuthal angle, respectively – and l is the topological charge of the beams. At the center, the phase has a screw dislocation, also called a phase singularity, or optical vortex. Typically, helical beams are formed by manipulating the light after it emerges from a laser by the superposition of two orthogonal (nonhelical) beams, or by transforming Gaussian beams into helical beams by means of computer-generated holograms (see Sacks, Rozas and Swartzlander [1998]), cylindrical lenses or spiral phase elements (SPEs) (Beijersbergen, Coerwinkel, Kristensen and Woerdman [1994]). Alternatively, a helical beam can be generated inside a laser cavity by inserting SPEs into the laser cavity (Oron, Davidson, Friesem and Hasman [2001]). The common approaches to forming SPEs are as refractive or diffractive optical elements using a milling tool, a single-stage etching process with a gray-scale mask, or a multistage etching process (Oron, Davidson, Friesem and Hasman [2001]). In general, such helical beam formations either are cumbersome or suffer from complicated realization, high aberrations, low efficiency, or large and unstable setups.

Biener, Niv, Kleiner and Hasman [2002] considered spiral phase elements based on the space-domain Pancharatnam–Berry phase. They showed that such elements could be realized using continuous computer-generated space-variant sub-wavelength dielectric gratings. Moreover, they experimentally demonstrated SPEs with different topological charges, based on Pancharatnam–Berry phase manipulation, with an axially symmetric local sub-wavelength groove orientation, for CO<sub>2</sub> laser radiation at a wavelength of 10.6 µm.

To design a PBOE with a spiral geometrical phase, we need to ensure that the direction of the grating grooves is given by  $\theta(r, \omega) = l\omega/2$ . By following the design procedure given in Section 3.1 the grating function,  $\phi_g$ , would result in

$$\phi_g(r,\omega) = \begin{cases} (2\pi r_0/\Lambda_0)(r_0/r)^{l/2-1}\cos[(l/2-1)\omega]/[l/2-1] & \text{for } l \neq 2, \\ \phi_g(r,\omega) = (2\pi r_0/\Lambda_0)\ln(r/r_0) & \text{for } l = 2, \end{cases}$$
(3.5)

where  $\Lambda_0$  is the local sub-wavelength period at  $r = r_0$ . For convenience, we use polar coordinates in the design procedure instead of the Cartesian coordinates used in Section 3.1. Biener, Niv, Kleiner and Hasman [2002] realized a Lee-type



Fig. 16. (a) Interferogram measurements of the spiral PBOEs. (b) The corresponding spiral phases for different topological charges. (From Biener, Niv, Kleiner and Hasman [2002].)

binary grating describing the grating function, given by eq. (3.5), for l = 1, 2, 3, 4. The grating was fabricated for CO<sub>2</sub> laser radiation with a wavelength of 10.6 µm. The geometry of the gratings for different topological charges are identical to those presented in fig. 4 in Section 2.1.3. The elements were fabricated on 500 µm thick GaAs wafers using contact photolithography.

Following the fabrication, the spiral PBOEs were illuminated with a right-hand circularly polarized beam,  $|\mathbf{R}\rangle$ , at 10.6 µm wavelength. In order to provide experimental evidence of the resulting spiral phase modification of their PBOEs, they used "self-interferogram" measurement using PBOEs with retardation of  $\phi = \frac{1}{2}\pi$ . For such elements, the transmitted beam comprises two different polarization orders:  $|\mathbf{R}\rangle$  polarization state, and  $|\mathbf{L}\rangle$  with phase modification of  $-l\omega$ , according to eq. (3.4). The near-field intensity distributions of the transmitted beams followed by a linear polarizer were then measured. Figure 16(a) shows the interferogram patterns for various spiral PBOEs. The dependence of the intensity on the azimuthal angle is of the form  $I \propto 1 + \cos(l\omega)$ , whereas the number of fringes is equal to l, the topological charge of the beam. Figure 16(b) illustrates the phase fronts resulting from the interferometer analysis, indicating spiral phases with different topological charges.

Figure 17 shows the far-field images of the transmitted beams through the spiral PBOEs with retardation  $\phi = \pi$ , having various topological charges, as well as the measured and theoretically calculated cross-sections. The experimental results were achieved by focusing the beams through a lens. Dark spots can be observed at the center of the far-field images, providing evidence of the phase singularity in the center of the helical beams. Excellent agreement between theory and experi-



Fig. 17. Experimental far-field images as well as their calculated and measured cross-sections for helical beams with l = 1-4. (From Biener, Niv, Kleiner and Hasman [2002].)

mental results was found, clearly indicating spiral phases for beams with different topological charges. In summary, we have demonstrated the formation of a helical wavefront using a PBOE, thereby proving the ability to form a complex geometrical phase using space-variant polarization manipulation.

### 3.2. Quantized Pancharatnam–Berry-phase diffractive optics

One of the most successful and viable outgrowths of holography involves diffractive optical elements (DOEs). These diffract light from a generalized grating structure having nonuniform groove spacing. They can be formed as thin optical elements that provide unique functions and configurations. High diffraction efficiencies for DOEs can be obtained with kinoforms that are constructed as surface-relief gratings on some substrate (d'Auria, Huignard, Roy and Spitz [1972]). However, in order to achieve a high efficiency, it is necessary resort to complex fabrication processes that provide the required accuracies for controlling the graded shape and depth of the surface grooves. Specifically, in a single process one photomask with variable optical density is exploited for controlling the etching rate of the substrate to form the desired graded relief gratings, or multiple binary photomasks are used so the graded shape is approximated by multilevel binary steps (see, for example, d'Auria, Huignard, Roy and Spitz [1972] and Dammann [1970]). Both fabrication processes rely mainly on etching techniques that are difficult to control accurately. As a result, the shape and depth of the grooves may differ from those desired, leading to reduced diffraction efficiency and poor repeatability of performance (Hasman, Davidson and Friesem [1991]).

Researchers have begun to investigate polarization diffraction gratings consisting of spatially rotating polarizers (Gori [1999]) or wave plates (Bhandari [1995]). Bomzon, Biener, Kleiner and Hasman [2002c] demonstrated simple polarization diffraction gratings based on continuous space-variant computer-generated subwavelength gratings. However, applying constraints on the continuity of the subwavelength grating leads to a space variation of the local period. As a result, the elements are restricted in their ability to form a desired complex phase function in addition to being limited in their physical dimensions. Moreover, the result of space-varying periodicity complicates the optimization of the photolithographic process.

In this subsection we present an approach for generating polarization-dependent DOEs based on quantized Pancharatnam–Berry-phase diffractive optics. Hasman, Kleiner, Biener and Niv [2003] have shown that such elements can be realized with a discrete geometrical phase, using a computer-generated space-variant sub-wavelength dielectric grating. By discretely controlling the local orientation of such grating, which has uniform periodicity, they were able to form more complex and sophisticated phase elements. They experimentally demonstrated quantized Pancharatnam–Berry-phase optical elements (QPBOEs) as a blazed diffraction grating and a polarization-dependent focusing lens, for the 10.6  $\mu$ m wavelength from a CO<sub>2</sub> laser. In addition, they showed that high diffraction efficiencies can be attained by utilizing a single binary computer-generated mask. This enabled the formation of multipurpose polarization-dependent optical elements that are suitable for applications such as optical interconnects, polarization beam splitting, optical switching and polarization-state measurements.

In the QPBOE approach, the continuous phase function  $\varphi_d(x, y)$  is approximated in discrete steps, leading to the formation of a PBOE with discrete local grating orientation. In the scalar approximation, an incident wavefront is multiplied by the phase function of the quantized phase element described by  $\exp[iF_N(\varphi_d)]$ , where  $\varphi_d$  is the desired phase and  $F_N(\varphi_d)$  is the actual quantized phase. The division of the desired phase  $\varphi_d$  into N equal steps is shown in fig. 18, where the actual quantized phase  $F_N(\varphi_d)$  is given as a function of the desired phase. The Fourier expansion of the actual phase front is given by  $\exp[iF_N(\varphi_d)] = \sum_p C_p \exp(ip\varphi_d)$ , where  $C_p$  is the *p*th-order coefficient of the Fourier expansion. The diffraction efficiency,  $\eta_p$ , of the *p*th-diffracted order is given by  $\eta_p = |C_p|^2$ . Consequently, the diffraction efficiency  $\eta_p$  for the first diffracted order for such an element is related to the number of discrete levels N by  $\eta_1 = [(N/\pi)\sin(\pi/N)]^2$ . This equation indicates that for 2, 4, 8, and 16 phase quantization levels, the diffraction efficiency will be 40.5, 81.1, 95.0, and 98.7%, respectively. The creation of a QPBOE is done by discrete orientation of the local sub-wavelength grating as illustrated in fig. 18.

One of the objectives of the study by Hasman, Kleiner, Biener and Niv [2003] was to design a blazed polarization diffraction grating, i.e., a grating for which all the diffracted energy is in the first order, when the incident beam is  $|\mathbf{R}\rangle$  polarized. They designed a QPBOE with retardation  $\phi = \pi$  that acted as a diffraction grating


Fig. 18. Actual quantized phase  $F(\varphi_d)$  as a function of the desired phase  $\varphi_d$ , as well as the discrete local grating orientation. Inset: scanning-electron microscope image of a region of the sub-wavelength structure of the focusing lens. (From Hasman, Kleiner, Biener and Niv [2003].)

by requiring that  $\varphi_d = (2\pi/d)x|_{\text{mod } 2\pi}$ . This formed the quantized phase function,  $F_N(\varphi_d)$ , depicted in fig. 18, where *d* is the period of the diffraction grating. In order to illustrate the effectiveness of their approach, they realized quantized diffraction gratings with various number of discrete levels, N = 2, 4, 8, 16, 128. The grating was fabricated for CO<sub>2</sub> laser radiation with a wavelength of  $\lambda = 10.6 \,\mu\text{m}$ , with the diffraction grating period  $d = 2.5 \,\text{mm}$  and the sub-wavelength grating period  $\Lambda = 2 \,\mu\text{m}$ . The dimensions of the elements were 30 mm × 3 mm and consisted of 12 grating periods. The magnified geometry of the grating for the case N = 4, and the predicted geometrical quantized phase distribution, are presented in fig. 19. The elements were fabricated on 500  $\mu$ m thick GaAs wafers using a single binary mask by means of contact photolithography. The insets in fig. 19 show scanning-electron microscopy images of some regions of the fabricated grating with a number of discrete levels, N = 4.

Following the fabrication, the QPBOEs were illuminated with a right-handed circularly polarized beam,  $|\mathbf{R}\rangle$ , at 10.6 µm wavelength. Figure 19 shows the measured and predicted diffraction efficiency for the first diffracted order for the different QPBOEs. The efficiencies are normalized relative to the total transmitted intensity for each element. The measured diffraction efficiency for N = 16 was  $99 \pm 1\%$ , the theoretical value being 98.7%. The excellent agreement between the experimental results and the predicted efficiency confirms the expected quantized phase.

In addition, Hasman, Kleiner, Biener and Niv [2003] formed a quantized Pancharatnam–Berry-phase focusing element for a 10.6 µm wavelength, having a



Fig. 19. Magnified geometry of the grating for N = 4, as well as the predicted geometrical quantized phase distribution, and scanning-electron microscopy images of some regions of the grating. Also shown are the measured (triangles) and predicted (dashed curve) diffraction efficiency as a function of the number of discrete levels, N. (From Hasman, Kleiner, Biener and Niv [2003].)

quantized spherical phase function of  $F_N(\varphi_d) = F_N[(2\pi/\lambda)(x^2 + y^2 + f^2)^{1/2}]$ with a diameter of 10 mm, a focal length of f = 200 mm, the number of discrete levels N = 8 and retardation  $\phi = \pi$ . Figure 20 illustrates the magnified geometry of a focusing lens based on a QPBOE with N = 4, as well as the predicted quantized geometrical phase. A scanning-electron microscope image of a region on the sub-wavelength structure that they had fabricated is shown in the inset of fig. 18. A diffraction-limited focused spot size for  $|L\rangle$  transmitted beam was measured, while illuminating the element with  $|\mathbf{R}\rangle$  polarization state. The inset in fig. 20 shows the image of the focused spot size as well as the measured and theoretically calculated cross-section. The measured diffraction efficiency was  $94.5 \pm 1\%$ , in agreement with the predicted value. The geometrical phase of a PBOE is polarization dependent, and this allowed them to experimentally confirm that their element is a converging lens for incident  $|\mathbf{R}\rangle$  state, and a diverging lens for incident  $|\mathbf{L}\rangle$  state, as indicated by eq. (3.4). For incident  $|\mathbf{L}\rangle$  state, the measured focal length was f = -200 mm as expected, and the measured diffraction efficiency was identical to the measured incident  $|\mathbf{R}\rangle$  state. Moreover, it is possible to form a bifocal lens as a PBOE with a retardation phase of  $\phi = \pi$  by illuminating with a linear polarization beam and inserting a refractive lens following the PBOE. A trifocal lens can also be created as a PBOE with a retardation phase of  $\phi = \frac{1}{2}\pi$ , resulting in three distinct focuses for  $|\mathbf{R}\rangle$ , linear, and  $|\mathbf{L}\rangle$  polarization states. As can be seen, the introduction of space-varying geometrical phases through QP-BOEs enables new approaches in the fabrication of polarization-sensitive optical elements.



Fig. 20. Illustration of the magnified geometry of a quantized-PBOE focusing lens with N = 4, as well as the predicted quantized geometrical phase. Inset: the image of the focused spot size as well as the measured (dots) and theoretically calculated (solid curve) cross-section. (From Hasman, Kleiner, Biener and Niv [2003].)

# 3.2.1. Propagation-invariant vectorial beams obtained by use of quantized Pancharatnam–Berry-phase optical elements

Propagation-invariant scalar fields have been extensively studied, both theoretically and experimentally, since they were first proposed by Durnin, Miceli and Eberly [1987]. These fields were employed in applications such as optical tweezers and the transport and guiding of microspheres (Garcés-Chávez, McGloin, Melville, Sibbett and Dholakia [2002]). While recently there has been considerable theoretical interest in propagation-invariant vectorial beams (Tervo and Turunen [2001]), experimental studies of such beams have remained somewhat limited (see Pääkkönen, Tervo, Vahimaa, Turunen and Gori [2002] or Bomzon, Niv, Biener, Kleiner and Hasman [2002a]). One of the most interesting types of propagation-invariant vectorial beams is the linearly polarized axially symmetric beam (LPASB) (see, for example, Niv, Biener, Kleiner and Hasman [2003]). These vectorial beams are characterized by their polarization orientation,  $\psi(\omega) = m\omega + \psi_0$ , where *m* is the polarization order number,  $\omega$  is the azimuthal angle of the polar coordinates, and  $\psi_0$  is the initial polarization orientation for  $\omega = 0$ . Methods for forming LPASBs have been discussed extensively in Section 2.

In this subsection we propose the formation of propagation-invariant vectorial Bessel beams by the use of QPBOEs followed by an axicon. Niv, Biener, Kleiner and Hasman [2004] demonstrated the formation of LPASBs with different polarization order numbers by using QPBOEs. They realized the QPBOEs by using computer-generated space-variant sub-wavelength gratings upon GaAs wafers for 10.6 µm laser radiation. The optical performance of the elements was experimentally evaluated by measuring the polarization distribution of the emerging beam through the QPBOE, verifying high quality LPASBs. Subsequently, propagation-invariant vectorial Bessel beams were achieved by inserting an axicon after the QPBOEs. As a final step, the resulting beams were transmitted through a polarizer which produced a unique propagation-invariant scalar beam. This beam had a propeller-shaped intensity pattern that could be rotated by simply rotating the polarizer, which makes it suitable for optical tweezers (MacDonald, Paterson, Volke-Sepulveda, Arlt, Sibbett and Dholakia [2002]).

The Jones vector of a LPASB is given by

$$|\mathbf{P}_{m}\rangle = \left[\exp(\mathrm{i}m\omega)|\mathbf{R}\rangle + \exp(-\mathrm{i}m\omega)|\mathbf{L}\rangle\right]/\sqrt{2},\tag{3.6}$$

where the  $|\mathbf{P}_m\rangle$  state represents the linearly polarized beam whose polarization azimuthal angle is given by  $\psi = m\omega$  (let us choose the reference axis so that  $\psi = 0$  at  $\omega = 0$ ). Propagation of the  $|\mathbf{P}_m\rangle$  state when transmitted through an axicon can be approximated by the stationary phase method (Pääkkönen, Tervo, Vahimaa, Turunen and Gori [2002]) to yield

$$|\mathbf{B}_{m}\rangle = \mathbf{K}_{z} \{ f_{a}(r) |\mathbf{P}_{m}\rangle \}$$
  

$$\cong (\pi \gamma \sqrt{z/\lambda}) \exp(ik [(1 - \gamma^{2}/2)z + r^{2}/2z - \lambda/8])$$
  

$$\times (-i)^{m} J_{m}(k\gamma r) |\mathbf{P}_{m}\rangle, \qquad (3.7)$$

where  $K_z$  is the Fresnel free space propagation operator for propagation distance *z*, *r* is the radial polar coordinate, *k* is the wave number, and  $J_m$  is the *m*th-order Bessel function of the first kind. In this case, the axicon phase function is paraxially approximated by  $f_a(r) = \exp(-ik\gamma r)$ , where  $\gamma = \beta(n-1)$  and  $\beta$  and *n* are the inclination angle and refractive index of the axicon, respectively. This paraxial calculation confirms propagation invariance of the polarization state as well as the Bessel intensity distribution, except for a linear growth function of z that can be removed by appodizing the incoming intensity (Davidson, Friesem and Hasman [1992b]). For this vectorial Bessel beam, the intensity profile is determined by m, the polarization order number of the original LPASB, while the local polarization state is unchanged by the axicon.

When illuminating a linearly polarized beam upon a QPBOE with retardation of  $\pi$  radians, the Jones vector of the emerging beam will be

$$|\mathbf{E}_{\text{out}}\rangle = \frac{1}{\sqrt{2}} \exp[\mathrm{i}2\theta(r,\omega)] |\mathbf{R}\rangle + \frac{1}{\sqrt{2}} \exp[-\mathrm{i}2\theta(r,\omega)] |\mathbf{L}\rangle.$$
(3.8)

According to eq. (3.8), the emerging beam,  $|\mathbf{E}_{out}\rangle$ , comprises two scalar waves of orthogonal circular polarizations, as expected from eq. (3.4). By selecting a local sub-wavelength groove orientation such as  $\theta = \frac{1}{2}m\omega$ , eq. (3.8) will be identical to eq. (3.6), and thus the desired  $|\mathbf{P}_m\rangle$  state will be obtained.

LPASBs with polarization order numbers m = 1, 2, 3 and 4 were formed by use of QPBOEs, as computer-generated space-variant sub-wavelength gratings. These elements were illuminated with a linearly polarized plane wave at a wavelength of 10.6 µm from a CO<sub>2</sub> laser. Scanning-electron microscope images of the elements' central sections are provided in fig. 21(a) for elements with polarization order numbers m = 2, 3. The local azimuthal angle was observed by inserting a polarizer directly behind the QPBOEs. The resulting intensities are depicted in fig. 21(b) for polarization order numbers m = 2, 3. Note that a specific azimuthal angle returns 2m times within each trip around the beam axis. Propagation-invariant vectorial beams were obtained by inserting a ZnSe axicon  $(\beta = 3^{\circ}, n = 2.4)$  following the QPBOEs. Figure 21(c) shows the intensities at 8 cm beyond the axicon for beams of polarization order number m = 2, 3. The double arrows, arranged along the circumference of the beams, illustrate the local azimuthal angles. The space-variant polarization state of the propagating beams was measured at different distances, verifying the vectorial propagation invariance of the beams.

Finally, the ability to achieve a controlled rotation of the intensity pattern by inserting a polarizer behind the axicon was demonstrated. It can be shown, again using stationary phase approximation, that transmittance of propagation-invariant LPASBs through a polarizer results in an amplitude of  $\propto J_m(k\gamma r) \cos(m\omega)$ . This beam is propagation-invariant with a propeller-shaped intensity pattern given by  $I \propto J_m^2(k\gamma r)(1 + \cos(2m\omega))$ . These propeller-shaped intensities are depicted in fig. 22(a). If the polarizer is rotated by an angle of  $\omega_0$ , the fringes rotate by an angle of  $\omega_0/m$ . This behavior is demonstrated in fig. 22(b), where the polarizer

(a) (b) (C) m=2m=3

Fig. 21. (a) Scanning-electron microscope images of the central parts of the OPBOEs for m = 2, 3 polarization orders. (b) Experimental intensity distributions, directly after the element, for beams emerging from the linear polarizer for m = 2, 3. (c) Intensity distributions at 8 cm beyond the QPBOE followed by an axicon for m = 2, 3. The strips arranged along the circumference of the beam illustrate the local azimuthal angles.

was rotated by 90°. The dashed and dotted lines indicate the resulting rotation of the propellers. It can be seen that rotations of  $90^{\circ}$ ,  $45^{\circ}$ ,  $30^{\circ}$  and  $22.5^{\circ}$  were obtained for m = 1, 2, 3 and 4, respectively.

# 3.3. Polarization Talbot self-imaging

The Talbot effect is a well-known interference phenomenon in which coherent illumination of a periodic structure gives rise to a series of self-images at welldefined planes (Talbot [1836]). This effect has many applications to fields such as wavefront sensing (Siegel, Loewenthal and Balmer [2001]), spectrometry (Kung, Bhatnagar and Miller [2001]) and Talbot laser resonators (Wrage, Glas, Fischer, Leitner, Vysotsky and Napartovich [2000]). Although most studies of the Talbot effect relate to waves for which the polarization is uniform, several contemporary papers have dealt with the Talbot effect in fields with space-variant polarization (see Arrizón, Tepichin, Ortiz-Gutierrez and Lohmann [1996] or Rabal, Furlan



4, § 3]

(a) (b) (b) (b) (c) (c)

Fig. 22. (a) Propeller-shaped intensity patterns of the beams emerging from the QPBOEs followed by an axicon and a polarizer for four polarization orders, m = 1, 2, 3, 4, from left to right. (b) Controlled rotation of the propeller-shaped intensities by rotating the polarizer by 90°; the dashed lines and the dotted curves indicate the rotation angles of the patterns. (From Niv, Biener, Kleiner and Hasman [2004].)

and Sicre [1986]). However, the experimental discussions were usually limited to simple binary anisotropic gratings or other discontinuous polarization distributions.

Arrizón, Tepichin, Ortiz-Gutierrez and Lohmann [1996] have shown that an anisotropic grating with two alternate linear perpendicular states of polarization is transformed by free propagation at one fourth of the Talbot distance into another grating with a circular polarization state. They formed this anisotropic grating by superposing two Ronchi-type diffraction gratings with a relative shift of half of the period, and with different polarization states. Figure 23 schematizes the theoretical space-variant polarization state at the near-field of the grating and at the quarter of the Talbot distance.

Further, in this section, we demonstrate a Talbot effect involving a PBOE for which the orientation of the fast-axis varies linearly in the *x*-direction. We show that for any incident polarization the resulting field undergoes self-imaging and fractional Talbot effects involving polarization, intensity and phase. Bomzon, Niv, Biener, Kleiner and Hasman [2002b] presented a theoretical analysis of the phenomenon and experimentally demonstrated the effect using a continuous space-variant sub-wavelength dielectric structure designed for  $CO_2$  laser radiation at a

258



Fig. 23. (a) Ronchi grating with period *d*, (b) anisotropic grating with linear polarization, and (c) anisotropic grating with circular polarization state, observed at the distance  $\frac{1}{4}Z_T$  behind the grating in (b). (From Arrizón, Tepichin, Ortiz-Gutierrez and Lohmann [1996].)

wavelength of 10.6 µm. Moreover, when a circularly polarized beam is incident upon the proposed PBOE a one-dimensional nondiffracting effect occurs, thus the beam emerging from the PBOE conserves its space-varying polarization and intensity as it propagates.

Let us assume a PBOE with  $t_x = t_y = 1$  and local grating orientation function  $\theta = \pi x/d|_{\text{mod }\pi}$ , where *d* is the rotation period of the space-variant wave plates' fast axis. In this case the emerging field can be calculated by using eq. (3.4). To prove that the emerging field,  $|\mathbf{E}_{\text{out}}\rangle$ , undergoes self-imaging, we calculate the propagation of each of the diffracted orders using the Fresnel approximation (see, for example, Goodman [1996]) to yield,

$$\left|\mathbf{E}_{\text{out}}(x,z)\right\rangle = \left\{\cos\frac{\phi}{2}|\mathbf{E}_{\text{in}}\rangle - i\sin\frac{\phi}{2}\left[\eta_{L}|\mathbf{L}\rangle\exp\left(-\frac{i2\pi x}{d} - \frac{i\pi\lambda z}{d^{2}}\right)\right. \\ \left. + \eta_{R}|\mathbf{R}\rangle\exp\left(\frac{i2\pi x}{d} - \frac{i\pi\lambda z}{d^{2}}\right)\right]\right\}\exp\left(\frac{i2\pi z}{\lambda}\right), \tag{3.9}$$

from which we find that  $|\mathbf{E}_{out}(x, z = 0)\rangle = |\mathbf{E}_{out}(x, z = mZ_T)\rangle$ , where z = 0 corresponds to the plane just after the grating,  $Z_T = 2d^2/\lambda$  is the Talbot distance, and *m* is an integer. This proves that  $|\mathbf{E}_{out}(x, z = 0)\rangle$  is reconstructed at the Talbot planes. Further analysis shows that  $|\mathbf{E}_{out}(x, z = \frac{1}{2}Z_T)\rangle = |\mathbf{E}_{out}(x + \frac{1}{2}d, z = 0)\rangle$ . Thus, at half the Talbot distance the field is shifted in the *x*-direction by half a period compared to the field at z = 0, demonstrating a fractional Talbot effect. We can expect additional interesting effects at other fractional Talbot planes. Figure 24 presents the concept of the vectorial Talbot effect along with the cal-



Fig. 24. Diffraction from the PBOE. The Talbot effect occurs in the region where the diffracted polarization orders overlap (the striped region). The polarization state at planes z = 0,  $\frac{1}{4}Z_T$ ,  $\frac{1}{2}Z_T$  and  $Z_T$ are also depicted for linearly polarized incident beam and retardation phase of the grating,  $\phi = \frac{1}{2}\pi$ . The planes are symbolized by the dashed lines in the striped region.

culated space-variant polarization state at the planes z = 0,  $\frac{1}{4}Z_T$ ,  $\frac{1}{2}Z_T$  and  $Z_T$ , for a linearly polarized incident beam, and using grating with retardation phase  $\phi = \frac{1}{2}\pi$ .

Bomzon, Niv, Biener, Kleiner and Hasman [2002b] have used a grating similar to that one presented in Section 3.1. They illuminated the elements with linearly polarized light and measured the Stokes parameters at various planes along the *z*-axis using the four-measurement technique (see Section 4 or Collett [1993] for a detailed discussion of the Stokes parameters). The experimental results agree with the predictions. At z = 0 just after the grating, the polarization varies periodically and continuously in the *x*-direction from linear polarization to nearly circular polarization, and the intensity is constant. This field is reconstructed at  $z = Z_T$ , thereby demonstrating the Talbot effect. At the plane  $z = \frac{1}{2}Z_T$  a shifted field can be observed as predicted by eq. (3.9). A fractional Talbot effect is also demonstrated at  $z = \frac{1}{4}Z_T$ . At this plane a clear periodic variation in intensity is observable. Although the polarization at this plane is space varying, the ellipticity is zero and the beam is linearly polarized at all points.

A case of special interest occurs when PBOE is illuminated with off-axis circularly polarized light at a small incident angle of  $\zeta \approx \lambda/2d$ . Using eqs. (3.4) and (3.9) we find that the resulting field when  $\phi = \frac{1}{2}\pi$  is

$$|\mathbf{E}_{out}(x,z)\rangle = \left[-\cos\left(\frac{2\pi x}{d} + \frac{\pi}{4}\right)\hat{\mathbf{x}} + \sin\left(\frac{2\pi x}{d} + \frac{\pi}{4}\right)\hat{\mathbf{y}}\right] \exp\left[i\left(\frac{2\pi z}{\lambda} + \frac{\pi}{4}\right)\right],$$



Fig. 25. Illustration of a PBOE for forming propagation-invariant fields. The insets show (top) the geometry of the PBOE, as well as (bottom) the vector-field formed by it. (From Bomzon, Niv, Biener, Kleiner and Hasman [2002a].)

where  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  are Cartesian unit vectors transverse to the direction of propagation. The resultant beam has uniform intensity and a constant space-variant polarization that is retained throughout its propagation. The beam is essentially a one-dimensional vectorial nondiffracting beam, analogous to a scalar nondiffracting cosine beam. Bomzon, Niv, Biener, Kleiner and Hasman [2002a] formed a vectorial nondiffracting beam by using a PBOE based on computer-generated space-variant sub-wavelength metal-stripe grating. Figure 25 illustrates a nondiffracting periodically space-variant polarization beam by using sub-wavelength gratings with  $\phi = \frac{1}{2}\pi$ . The transmitted beam comprises two polarization orders which travel in different directions. The interference of the two polarization orders in the region where they overlap results in a propagation-invariant beam. The uniqueness of the vectorial solution lies in its space-varying polarization and uniform intensity, which makes it better suited for applications such as metrology and three-dimensional scanning.

#### §4. Applications of space-variant polarization manipulation

Space-variant polarization-state manipulation has been found useful in a wide range of research fields. Quabis, Dorn, Eberler, Glöckl and Leuchs [2000] showed theoretically that the focal area is reduced when a radially polarized instead of a linearly polarized light annulus is used. This reduction of the focal area can be utilized for improving the spatial resolution in imaging systems. Liu, Cline and

He [1999] used a radially polarized, high-power, Gaussian  $CO_2$  laser beam interacting with a high-quality electron beam located at the Brookhaven accelerator test facility, in order to perform laser acceleration in vacuum. They noted that a radially polarized, Gaussian laser beam can produce a stronger longitudinal electric field than a linearly polarized one. Niziev and Nesterov [1999] investigated the influence of the beam's polarization on laser cutting efficiency. They concluded that in the case of cutting metals with a large ratio of sheet thickness to width of cut, the laser cutting efficiency for radially polarized beams is 1.5-2 times higher than for plane *p*-polarization (TM-polarization) and circularly polarized beams.

In this section we elaborate on several applications using space-variant polarization manipulation performed by polarization-dependent optical elements such as polarization gratings. A polarization grating is defined, according to Gori [1999], as a transparency in which the polarization of the incident wave is changed periodically. In Section 4.1 we focus on polarization measurements, including near-field polarimetry and far-field polarimetry by use of spatially varying polarization manipulation and imaging polarimetry. In Section 4.2 we review depolarization methods with special attention to depolarizers based on space-variant polarization manipulation. Section 4.3 discusses the application of polarization encryption, emphasizing geometrical phase encryption, and polarization encoding, as well as optical computing as an example of an interesting encoding application. Section 4.4 describes the possibility of spatial control of polarization-dependent emissivity using sub-wavelength-structured elements.

# 4.1. Polarization measurements

In this subsection we discuss polarization measurements performed by spacevariant polarization-state manipulation. Optical polarization measurement has been widely used for a wide range of applications such as ellipsometry (Lee, Koh and Collins [2000]), bio-imaging (Sankaran, Everett, Maitland and Walsh [1999]), imaging polarimetry (Nordin, Meier, Deguzman and Jones [1999]), and optical communications (Chou, Fini and Haus [2001]). A commonly used method is to measure the time-dependent signal once the beam has been transmitted through a photoelastic modulator (see Jellison [1987]) or a rotating quarter-wave plate (QWP) followed by a polarizer–analyzer (see, for example, Collett [1993]). The polarization state of the beam can be derived by Fourier analysis of the detected signal. An increasing demand for faster and simpler methods has led to the development of the simultaneous four-channel ellipsometer (Azzam [1987]). Oka and Kato [1999] reported on a method for spectroscopic measurement of the spectrally resolved polarization state. In their scheme, the light is passed successively through a pair of thick birefringent retarders and a polarizer–analyzer. The light emerging from the polarizer–analyzer is then fed into a spectrometer followed by a photodetector. Finally, the signal obtained by the photodetector is analyzed by a computer. The light that is being measured is assumed to have a broad-band spectrum.

# 4.1.1. Near-field polarimetry

Bomzon, Biener, Kleiner and Hasman [2002a] presented a Fourier-transform polarimeter that is a space-domain analogue to the rotating QWP polarimeter method using a continuous space-variant dielectric sub-wavelength grating. Biener, Niv, Kleiner and Hasman [2003b] and Hasman, Biener, Kleiner and Niv [2003] later proposed a Fourier-transform polarimeter using discrete spacevariant sub-wavelength dielectric gratings. The grating of this type of element is divided into equal-sized zones. The sub-wavelength grooves are of uniform orientation and period within each zone and are rotated at discrete angles from zone to zone. The measurements for this type of polarimeter are performed in the near field, therefore this polarimeter is referred to as a near-field polarimeter. A Fourier-transform polarimeter using discrete space-variant sub-wavelength dielectric gratings is less sensitive to statistical errors because of the increased number of measurements, it is suitable for real-time applications, and it can be used in compact configurations. In addition, it is possible to integrate this polarimeter on a two-dimensional detector array for lab-on-chip applications. The high throughput achieved and the low cost make it useful as a commercial polarimeter for biosensing.

The concept of near-field polarimetry based on sub-wavelength gratings is presented in fig. 26. Uniformly polarized light is incident upon a polarization- sensitive medium (e.g., biological tissue, an optical fiber, a wave plate, etc.) and then transmitted through a space-variant sub-wavelength grating that acts as a space-variant wave plate, followed by a polarizer. The space-variant wave-plate element is a particular case of polarization grating. The resulting intensity distribution is detected by a camera and captured for further analysis. The emerging intensity distribution is uniquely related to the polarization state of the incoming beam. This dependence is given by a spatial Fourier-series analysis, wherein the resulting Fourier coefficients completely determine the polarization state of the incoming beam.

The polarization state within the Stokes representation is described by a Stokes vector  $\mathbf{S} = (S_0, S_1, S_2, S_3)^T$ , where  $S_0$  is the intensity of the beam, and  $S_1, S_2$ ,  $S_3$  represent the polarization state. In general,  $S_0^2 \ge S_1^2 + S_2^2 + S_3^2$ , where the equality holds for fully polarized beams. The degree of polarization (DOP) of a



Fig. 26. Schematic presentation of near-field spatial Fourier-transform polarimetry based on a discrete space-variant sub-wavelength dielectric grating followed by a sub-wavelength metal polarizer. Also shown is the measured intensity distribution captured in a single camera frame when the fast axis of the rotating QWP was at an angle of 20°. (We used a CO<sub>2</sub> laser that emitted linearly polarized light and replaced the polarization-sensitive medium with a rotating QWP.) (From Hasman, Biener, Kleiner and Niv [2003].)

beam is defined by DOP =  $(S_1^2 + S_2^2 + S_3^2)^{1/2}/S_0$ . The polarization state emerging from an optical system (e.g., wave plates, polarizers, etc.) is linearly related to the incoming polarization state through  $\mathbf{S}' = \mathbf{MS}$ , where  $\mathbf{M}$  is a 4-by-4 real Mueller matrix of the system and  $\mathbf{S}$  and  $\mathbf{S}'$  are the Stokes vectors of the incoming and outgoing polarization states, respectively (see Collett [1993] for further reading). The optical system under consideration consists of a polarization grating followed by a polarizer. This composite element can be described in Cartesian coordinates by a Mueller matrix

$$\mathbf{M}(\theta) = \mathbf{M}_{P}\mathbf{M}_{R}(-\theta)\mathbf{M}_{WP}(\phi)\mathbf{M}_{R}(\theta), \qquad (4.1)$$

where  $\theta$  represents the discrete rotation angle of the retarder (e.g., sub-wavelength dielectric grating) as a function of its location along the *x*-axis,

$$\mathbf{M}_{R}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & \cos 2\theta & \sin 2\theta & 0\\ 0 & -\sin 2\theta & \cos 2\theta & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$
(4.2)

is the Mueller matrix that represents rotation of the axis frame by angle  $\theta(x)$ ,

$$\mathbf{M}_{WP}(\phi) = \frac{1}{2} \begin{pmatrix} t_x^2 + t_y^2 & t_x^2 - t_y^2 & 0 & 0\\ t_x^2 - t_y^2 & t_x^2 + t_y^2 & 0 & 0\\ 0 & 0 & 2t_x t_y \cos \phi & -2t_x t_y \sin \phi\\ 0 & 0 & 2t_x t_y \sin \phi & 2t_x t_y \cos \phi \end{pmatrix}$$
(4.3)

is the Mueller matrix of a transversally uniform retarder, with retardation  $\phi$  and real transmission coefficients for two eigen-polarizations  $t_x$  and  $t_y$ , and

is the Mueller matrix of an ideal horizontal polarizer.

The outgoing intensity can be related to the incoming polarization state of the beam by calculating the Mueller matrix given above and using the linear relation between the incoming and the outgoing Stokes vectors, yielding

$$S'_{0}(x) = \frac{1}{4} \left\{ AS_{0} + \frac{1}{2}(A+C)S_{1} + B(S_{1}+S_{0})\cos 2\theta(x) + (BS_{2}-DS_{3})\sin 2\theta(x) + \frac{1}{2}(A-C)[S_{1}\cos 4\theta(x) + S_{2}\sin 4\theta(x)] \right\},$$
(4.5)

where  $A = t_x^2 + t_y^2$ ,  $B = t_x^2 - t_y^2$ ,  $C = 2t_x t_y \cos \phi$ , and  $D = 2t_x t_y \sin \phi$ . Equation (4.5) describes the intensity of the outgoing beam as a truncated Fourier series with coefficients that depend on the Stokes parameters of the incident beam.  $S_0-S_3$  would be extracted using Fourier analysis (for a detailed discussion see Biener, Niv, Kleiner and Hasman [2003b]). We note that the grating coefficients A, B, C and D should be determined by direct measurement of the polarization grating parameters,  $t_x$ ,  $t_y$ , and  $\phi$ , or by performing a suitable calibration process.

Biener, Niv, Kleiner and Hasman [2003b] fabricated a discrete space-variant sub-wavelength grating element for CO<sub>2</sub> laser radiation of 10.6  $\mu$ m wavelength. The dimensions of the element were 30 mm × 3 mm and consisted of 12 periods of *d*. They used the setup shown in fig. 26 to demonstrate experimentally the ability of their method to measure the polarization state for fully and partially polarized light (see figs. 27 and 28). The ability of their device to conduct polarization measurements of fully polarized light was tested by using a CO<sub>2</sub> laser that emitted linearly polarized light and replacing the polarization-sensitive medium with a rotating QWP. Figure 27 shows the experimental and theoretical azimuthal



Fig. 27. Measured (circles) and predicted (solid curves) values for azimuthal and elipticity angles as a function of orientation of the QWP. (From Hasman, Biener, Kleiner and Niv [2003].)



Fig. 28. Calculated (solid curve) and measured (circles) DOP as a function of the intensity ratio of two independent lasers having orthogonal linear polarization states, as used in the setup depicted in the top inset. The bottom inset shows calculated (solid curves) and measured (circles) intensity cross-sections for two extremes,  $I_1 = I_2$  (DOP = 0.059) and  $I_2 = 0$  (DOP = 0.975). (From Hasman, Biener, Kleiner and Niv [2003].)

angle,  $\psi$ , and the ellipticity  $\chi$ , calculated from the measured data, by use of the relations  $\tan(2\psi) = S_2/S_1$  and  $\sin(2\chi) = S_3/S_0$ . The partially polarized beam was constructed by combining two CO<sub>2</sub> lasers with orthogonal polarization (the setup is shown in the inset of fig. 28). Figure 28 shows the measured and predicted DOP as a function of the intensity ratio,  $I_1/I_2$ , of the combined lasers. The inset shows the experimental intensity distributions for two extreme cases. The first is for equal intensities  $(I_1 = I_2)$ , in which the measured DOP is 0.059, indicating unpolarized light. The second is for illumination by a single laser only (i.e.,  $I_2 = 0$ ), in which the measured DOP is 0.975, indicating fully polarized light.



Fig. 29. Experimental interferograms generated by the OBPI filter of fig. 1. Alongside each interference pattern is the associated incident Stokes vector. A 50 µm scale bar is shown in the lower right-hand corner of each image. (From Van Delden [2003].)

Van Delden [2003] proposed an interferometric approach to polarization measurement by the use of an ortho-Babinet, polarization-interrogating filter (OBPI). His polarimeter is composed of four birefringent wedges and a linear polarizer comprising an OBPI filter assembly. Operationally, the OBPI filter is characterized by a three-stage optical system consisting of two modified Babinet compensators (i.e., spatially varying linear retarders) and a linear polarizer. The most important feature of the resulting interferogram is that a unique pattern is generated for any polarization state of the incident beam. Figure 29 shows experimental interferograms produced by the assembled OBPI filter with varying conditions of the polarized Koehler illumination.

### 4.1.2. Far-field polarimetry

Gori [1999] proposed measuring the Stokes parameters by means of a polarization grating comprised of a linear polarizer whose orientation varied periodically along a line. His analysis was done using Jones calculus. A more general case of polarization grating of a periodically rotating wave plate analyzed using Jones calculus was presented in Section 3. The analysis of a beam emerging from such a polarization grating is given by eq. (3.4). As noted in Section 3, the beam emerging from a polarization grating comprises three polarization orders. The first maintains the original polarization state and phase of the incident beam, the second

is right-handed circularly polarized and has a phase modification of  $2\theta(x)$ , and the third has a polarization direction and phase modification opposite to those of the former polarization order. Gori assumed  $\theta(x)$  to be a linear function of x, in which case the second and third polarization orders become 1 and -1 diffraction orders, respectively, and the first polarization order becomes the zeroth diffraction order. Gori showed that the decomposition performed by the polarization grating can be used for evaluating the Stokes parameters of a light beam. Suppose that the beam to be analyzed is sent through the polarization grating followed by a polarizer. At a suitable distance from the grating the three diffraction orders are spatially separated. By setting the polarizer orientation angle at the two values of 0° and 45° and measuring the corresponding intensities of the undiffracted-order beam together with the intensities of the 1 and -1 diffracted beams (at any angle of the polarizer) the Stokes parameters of the beam can be obtained. Figure 30 depicts the far-field intensities measured for a beam being passed through the polarization grating, a lens and a polarizer oriented at  $0^{\circ}$  and  $45^{\circ}$ . The polarization grating, which was realized by dielectric space-variant sub-wavelength gratings, was illuminated by a CO<sub>2</sub> laser at the wavelength of  $\lambda = 10.6 \,\mu\text{m}$ . The inset in fig. 30(a) shows a scanning-electron microscopy image of a region on the subwavelength structure on the GaAs wafer. The measurements were taken in the focal plane of the lens as illustrated in the inset of fig. 30(b). The polarization state of the measured incident beam was generated by transmitting a linearly polarized beam oriented at  $0^{\circ}$  through a QWP with its fast axis oriented at  $20^{\circ}$  with



Fig. 30. Cross-section of the measured intensity in the Fourier plane of a beam emerging from a spatially rotating QWP polarimeter followed by a polarizer oriented at (a) 0° and (b) 45°. The polarization of the incident beam was generated by illuminating a QWP oriented at 20° with a horizontal linear polarized CO<sub>2</sub> laser beam at the wavelength of  $\lambda = 10.6 \mu m$ . The inset depicted in (b) describes the concept of far-field polarimetry. The far-field polarimeter contains a discrete space-variant sub-wavelength dielectric polarization grating (D), a lens (L) and a polarizer (P). The inset in (a) is a scanning-electron microscope image of a region of the sub-wavelength structure.

respect to the *x*-axis. The measured azimuthal angle,  $\psi$ , and the ellipticity,  $\chi$ , of the incident beam were 16° and 20.7°, respectively, which is in good agreement with the predicted results.

### 4.1.3. Imaging polarimetry

A more general application of polarization measurement is imaging polarimetry, which is being investigated as a means to extend the capabilities of infrared (IR) systems beyond conventional amplitude imaging. For example, some polarization matrices used in imaging polarimetry offer the capability to highlight or suppress different materials in a scene, or objects in different orientations. As a result, imaging polarimetry offers a means to extend the capabilities of conventional IR imaging and to provide new imaging modalities. Nordin, Meier, Deguzman and Jones [1999] and Guo and Brady [2000] proposed a micropolarizer array for IR imaging polarimetry. The polarization-imaging camera proposed by Nordin, Meier, Deguzman and Jones [1999] consisted of a  $128 \times 128$  array of unit cells, each of which was composed of a  $2 \times 2$  array of sub-wavelength metal strips in different orientations that acts as a  $2 \times 2$  array of micropolarizers. Figure 31 depicts a unit cell within the micropolarizer array that contains two upper micropolarizers oriented at 90°, and two lower micropolarizers oriented at 0° and 45°. The unit cell illustrated in fig. 31 is considered to comprise a single image pixel. Oka and Kaneko [2003] proposed an alternative design for imaging polarimetry



Fig. 31. Schematic diagram of unit cell containing a 2 × 2 array of micropolarizers. (From Nordin, Meier, Deguzman and Jones [1999].)



Fig. 32. Configuration of the block of polarimetric devices. (From Oka and Kaneko [2003].)

as illustrated in fig. 32. Their instrument consisted of two pairs of birefringent wedge prisms cemented together and a polarizer–analyzer.

### 4.2. Spatial polarization scrambling

In this subsection we present methods for depolarizing light, based on spacedomain polarization-state scrambling. Depolarizers are optical elements that reduce the degree of polarization (DOP) of a beam, independent of its incident polarization state. These components are essential for removing undesired polarization sensitivity in optical systems, such as for long-haul transmission systems that use erbium-doped fiber amplifiers (Mazurczyk and Zyskind [1994]), and for optical measurement equipment (Kersey, Marrone and Dandridge [1990]). Totally unpolarized light is described by a Stokes vector of the form  $\langle \mathbf{S} \rangle = (\langle S_0 \rangle, 0, 0, 0)^T$ , where the angle brackets denote the average value over the space domain. Therefore, for a uniform incident beam, the components of the Mueller matrix of a perfect depolarizer,  $\langle \mathbf{M}_{dep} \rangle$ , are given by  $\langle m_{ij}^{dep} \rangle = 0$ , except for  $\langle m_{11}^{dep} \rangle = 1$ . Stokes–Mueller calculus has been described above, in Section 4.1.

Several approaches for depolarizing light based on the scrambling of the polarization state in the time or wavelength domains have been suggested and experimentally demonstrated. Lyot [1928] was the first to propose an approach for reducing the DOP of a beam. His method relies on polarization scrambling over the wavelength. Billings [1951] and Heismann and Tokuda [1995] proposed the possibility of depolarizing monochromatic beams using a temporally varying retarder.

McGuire and Chipman [1990] suggested a crystal-based depolarizer for scrambling the polarization state in the space domain. They suggested using two identical Babinet compensators oriented at 45° to each other. Each Babinet compensator, in turn, consisted of two prisms cemented together, one with the fast axis horizontal and the other with the fast axis vertical. Figure 33 shows the concept of the depolarizer using double Babinet compensators. Spatial polarization-state



Fig. 33. Construction of the dual Babinet compensator pseudodepolarizer. H and V denote birefringent material with the fast axis horizontal and vertical, respectively. (From McGuire and Chipman [1990].)

scramblers are compact, passive components, and suitable for use in real-time applications and with monochromatic laser radiation.

Biener, Niv, Kleiner and Hasman [2003a] proposed a complete depolarizer based on space-domain polarization-state scrambling performed by cascaded, computer-generated, space-variant sub-wavelength dielectric gratings, as shown in fig. 34(a). The first is a space-variant quarter-wave plate (QWP) with a rotation



Fig. 34. (a) Schematic presentation of our concept for depolarizing over the space domain. The insets illustrate the geometry of the sub-wavelength gratings. (b) SEM image of a typical cross-section of the grating profile of the QWP. (From Biener, Niv, Kleiner and Hasman [2003b].)



Fig. 35. Illustration of the outgoing beam's polarization state when the polarization of the incoming beam is (a) vertically linear, (b) horizontally linear, (c) linear at 45° or (d) circular. The spheres show the trajectories of the outgoing polarization states onto the Poincaré sphere. (From Biener, Niv, Kleiner and Hasman [2003b].)

period of  $d_1 = \frac{1}{4}d$ ; the second is a space-variant half-wave plate (HWP) with a rotation period of  $d_2 = d$ . The Mueller matrix of a wave plate of which the fast axis rotates periodically with respect to the position along the *x*-axis is described in Section 4.1 with  $\theta = \pi x/d$ , where *d* is the rotation period. Calculating the average value along the *x*-axis as  $\langle m_{ij} \rangle = (2/d) \int_0^{d/2} m_{ij}(x) dx$  yields the Mueller matrix of an ideal depolarizer. The depolarization effect is achieved by spatially scrambling the beam's polarization state. Figure 35 illustrates the local outgoing polarization state as a trajectory on a Poincaré sphere (see Brosseau [1998] for a detailed explanation on Poincaré spheres). As shown in fig. 35, the resulting space-variant polarization state includes polarization ellipses of different orientation and ellipticity. These polarization ellipses demonstrate the principle of the scrambling procedure.

Biener, Niv, Kleiner and Hasman [2003a] realized Lee-type gratings, which describe their grating functions. The first grating was a spatially rotating QWP with  $d_1 = 2.5$  mm; the second was a spatially rotating HWP with  $d_2 = 10$  mm. The elements were fabricated on a GaAs wafer. Figure 34(b) shows a scanning-electron microscope image of a typical cross-section of the grating profile of the QWP at a period of about 2 µm. The measured phase retardations of the elements were 0.46 $\pi$  and 0.96 $\pi$  for the appropriate QWP and HWP, respectively. The retardation of the elements was measured for 10.6 µm wavelength radiation. These



Fig. 36. Measured and predicted DOP as a function of the orientation of the QWP, through which the incident beam has been transmitted. (From Biener, Niv, Kleiner and Hasman [2003b].)

results are in good agreement with the theoretical predictions achieved by rigorous coupled-wave analysis, utilizing the measured profiles of the gratings.

Subsequently, their depolarizer was experimentally tested using linearly polarized CO<sub>2</sub> laser radiation at a wavelength of 10.6 µm. They illuminated a rotating QWP in order to manipulate the polarization state of the beam incident on the depolarizer. The polarization state of the beam emerging from the depolarizer was measured using the four-measurement technique (see Collett [1993]). Each measurement was obtained by summing the intensity over the *x*-axis over the interval  $0 < x < \frac{1}{2}d_2$ . Figure 36 shows the measured and predicted DOP as a function of the orientation of the QWP. The experimental DOP attained was less than 0.16.

When the polarization state of the incident beam is known, the use of a simple pseudo-depolarizer is sufficient. Biener, Niv, Kleiner and Hasman [2003a] have demonstrated that a single, spatially rotating QWP or HWP based on spacevariant sub-wavelength dielectric polarization gratings can completely depolarize incident light with circular and linear polarization states, respectively. They used the same sub-wavelength polarization grating previously described for the cascaded gratings. The experimentally measured DOPs for the QWP and HWP scramblers were 0.021 and 0.075, respectively.

#### 4.3. Polarization encryption and polarization encoding

In this subsection we present several concepts for polarization encryption as well as numerous methods for the polarization encoding of data by using space-variant polarization-state manipulation. In the past few years there has been increasing interest in data security and a growing need for improved methods for encrypting data. One of the processes that has been extensively investigated is the optical encryption technique. Different optical encryption schemes have been suggested, for example schemes involving pure amplitude image encryption suggested by



Fig. 37. Schematic diagram showing (a) the generic system architecture; and (b) the polarization encoding and decoding geometry in greater detail. The system operates as a direct phase-only mapping of the encrypted mask and decrypting key. A pair of crossed polarizers (P1 and P2) are used to generate an intensity pattern at the output. The polarization directions of the various components are indicated, the decrypting key (D) and encrypted mask (E) are aligned so that they act to phase-shift only one orthogonal component of the polarized wavefront (a). (From Mogensen and Glückstad [2000].)

Unnikrishnan, Joseph and Singh [1998]. Other encryption schemes involving phase-only images were explored by Towghi, Javidi and Luo [1999] in order to improve the visibility of the decrypted image. Both methods use double-random phase encryption, a technique first presented by Refregier and Javidi [1995].

Polarization encryption has been investigated by several groups, each employing slightly different concepts. Polarization encryption provides additional flexibility in the key encryption design by adding polarization-state manipulation to the conventional phase and amplitude manipulation used in earlier methods. This feature is advantageous as it makes the polarization encryption method more secure. Mogensen and Glückstad [2000] proposed polarization encryption using spatially modulated retardation. Their optical decryption system is shown schematically in fig. 37(a). The polarization procedure for encoding and decoding the phase information is shown in greater detail in fig. 37(b). A laser beam linearly polarized along the 45° axis is aligned with the polarizer P1. The encrypted phase mask (E) and the decrypting phase key (D) are aligned such that their fast axes are parallel to the *y*-axis. A second polarizer (P2) at an angle of 135°, crossed with respect to the first, is used to produce an intensity read-out of the phase-shifting information. The phase masks were implemented using a pair of parallel-aligned liquid-crystal spatial light modulators supplied by Hamamatsu Photonics.



Fig. 38. Schematic of a system for polarization encryption; P<sub>1</sub>, input polarizer; P<sub>2</sub>, output polarizer; L<sub>1</sub>, L<sub>2</sub>, imaging lenses. (From Unnikrishnan, Pohit and Singh [2000].)

A second scheme for polarization encryption was proposed by Unnikrishnan, Pohit and Singh [2000] using a ferroelectric liquid-crystal spatial light modulator. Their encryption is done by an exclusive-OR (XOR) operation between the image and a random phase code (a key used to encrypt the data). The XOR operation is carried out in the polarization domain of coherent light by using two ferroelectric liquid-crystal spatial light modulators. The decryption of the encrypted data is done by a second XOR operation between the encrypted image and the key. Figure 38 shows a schematic representation of the concept of polarization encryption using ferroelectric liquid-crystal spatial light modulator.

A different recording method for polarization encryption was suggested by Tan, Matoba, Okada-Shudo, Ide, Shimura and Kuroda [2001] using bacteriorhodopsin (a polarization recording medium). This method uses interference to record the spatially scrambled polarization field onto a medium that is sensitive to the electromagnetic field but not the intensity. Figure 39 shows the experimental setup using a polarization mask formed by polarizer films, a spatial light modulator for encrypting the polarization data, and a polarization recording medium (bacteriorhodopsin). The experimental setup shown in fig. 39 includes the decryption process.

A concept of double-random polarization encryption was suggested by Matoba and Javidi [2004]. Their method includes one polarization scrambling mask located at the image plane and a second polarization scrambling mask located at the Fourier plane. This is considered to be a more secure method for encrypting data due to the use of two scrambling keys. Figure 40 shows the schematic of the proposed double-random polarization encryption technique. A different concept of polarization encryption involving geometrical phase was proposed by Biener,



Fig. 39. Experimental setup for encryption using bacteriorhodopsin: SP, spatial filter; M, mirror; BS, beam splitter; P, polarizer; L, lens; CCD, CCD camera; RMM, random modulation mask; BR, bacteriorhodopsin; PC, personal computer. (From Tan, Matoba, Okada-Shudo, Ide, Shimura and Kuroda [2001].)



Fig. 40. Schematic of the double-random polarization encryption technique: *f*, focal length. (From Matoba and Javidi [2004].)

Niv, Kleiner and Hasman [2005]. The connection between geometrical phase and spatially varying polarization state manipulation was discussed in Section 3.

The concept of geometrical phase encryption involves a PBOE that encodes the image intensity added by a random key function. (The term PBOE is explained in detail in Section 3.) The proposed PBOE, which is a space-variant rotating wave plate, imprints the image intensity plus the random key function in the local orientation of the wave plate's fast axes. Let us assume that a PBOE with a space-varying wave-plate orientation function of  $\theta_i(x, y)$  encodes the primary image



Fig. 41. (a) Schematic representation of the concept of geometrical phase encryption. (b) Primary image intensity to be encrypted. (c) Encrypting PBOE wave plate's orientation function,  $\theta_i + \theta_k$ , in grayscale. (d) Sub-wavelength grating mask of the PBOE. (e) Polarization state of the beam emerging from the PBOE. (d) and (e) are taken from a small region near the eyebrow of Einstein, which is depicted in (b).

of young Einstein, depicted in fig. 41(b). In order to further encrypt the encoded primary image information embedded in the PBOE, we add a random rotation function,  $\theta_k(x, y)$ , to the space-varying wave plates' orientation. This random rotation factor serves as an encryption/decryption key. The total orientation function of the wave plates, comprising the encrypted PBOE, is shown in grayscale in fig. 41(c). Decryption is performed by illuminating the encrypted element with circularly polarized light and then analyzing the emerging Stokes parameters with the appropriate key to retrieve the primary image. The scheme for this process is shown in fig. 41(a).

The beam emerging from a PBOE, which is a rotating QWP, illuminated by  $|\mathbf{R}\rangle$ -polarized light comprises two polarization orders, as can be seen in eq. (3.4). The first maintains the original polarization state and phase of the incident beam, and the second is left-hand circularly polarized,  $|\mathbf{L}\rangle$ , and has a phase modification of  $-2\theta(x, y)$ . The phase added to the  $|\mathbf{L}\rangle$ -polarized beam, which is geometrical in nature, equals  $-(\varphi_i + \varphi_k)$ , where  $\varphi_i = 2\theta_i$  and  $\varphi_k = 2\theta_k$  denote the geometrical phase added by the encoded primary image's intensity and the encoded key re-



Fig. 42. (a–c) Three intensity pictures generated by the decryption process for the polarizer in the different orientations: (a)  $0^{\circ}$ , (b)  $45^{\circ}$  and (c)  $90^{\circ}$ . The arrows indicate the orientation angle of the polarizer. (d) Decrypted image achieved by the decryption process using the intensities shown in (a–c).

spectively. Figure 41(e) depicts the space-variant polarization direction emerging from a PBOE with optical parameters of  $t_x = t_y = 1$  and  $\phi = \frac{1}{2}\pi$ . The emerging field, which is a result of the vectorial self-interference, is a space-varying polarized field. As can be seen, the orientation of the arrows is random. In order to retrieve the primary image's geometrical phase we need to measure the Stokes parameters of the emerging beam. The Stokes parameters are measured by using a polarizer oriented in three different orientations. These measurements have been discussed extensively by Biener, Niv, Kleiner and Hasman [2005]. By using the measured Stokes parameters and by applying the geometrical phase key, we can retrieve the phase function of the primary image. For the realization of the optical concept, we can implement a method first discussed in Section 2 for space-variant polarization-state manipulations using computer-generated sub-wavelength structures. Figure 41(d) is a magnified illustration of the sub-wavelength grating mask of the encrypted element.

A computer simulation was performed to test the geometrical phase encryption. Figures 42(a)–42(c) show the three intensity pictures obtained by measuring the encrypted image after being transmitted through a simulated polarizer oriented at three different orientations. The decrypted image shown in fig. 42(d) was generated by calculating the Stokes parameters when applying the simulated intensities, and by applying the correct geometrical phase key,  $\varphi_k$ .

Polarization encryption can be considered to be a specialized form of polarization encoding, which is a general application of space-variant polarization-state manipulation. There are several methods for encoding the space-variant polarization state of a vectorial field. Eriksen, Mogensen and Glückstad [2001] and Davis, McNamara, Cottrell and Sonehara [2000] proposed the use of dynamic modulation of the space-variant polarization state by use of spatial light modulators. Figure 43 depicts the concept of polarization encoding by using spatial light modulators. Figure 44 shows the image produced by polarization encoding using



Fig. 43. An optical system for converting incident polarized light into an arbitrary state of elliptically polarized light with the major axis of the elliptically polarized light rotated by an arbitrary angle. The lines denote the extraordinary axis of the SLMs, the quarter-wave plates  $(\frac{1}{4}\lambda)$  and the polarization direction of the linear polarizer. (From Eriksen, Mogensen and Glückstad [2001].)



Fig. 44. (a) Intensity pattern with the analyzer–polarizer perpendicular to the input polarizer. (b) Intensity pattern with the analyzer–polarizer parallel to the input polarizer. (c) Intensity pattern with the analyzer–polarizer parallel to the ordinary axis of the LCSLM. (d) Intensity pattern with the analyzer–polarizer parallel to the extraordinary axis of the LCSLM. (From Davis, McNamara, Cottrell and Sonehara [2000].)

a polarizer–analyzer in different orientations. Another approach, discussed extensively throughout this chapter, is utilizing space-variant sub-wavelength gratings. Zeitner, Schnabel, Kley and Wyrowski [1999] demonstrated diffractive elements with polarization multiplexing for visible light constructed with metal-stripe sub-



Fig. 45. Scanning-electron microscope picture of the element structure etched into the chromium layer. (From Zeitner, Schnabel, Kley and Wyrowski [1999].)

wavelength period gratings. They introduced different functions of the element for two orthogonal polarization directions using polarization-dependent pixel transmission, which was realized by sub-wavelength gratings within a pixel. Figure 45 shows a scanning-electron microscope picture of a region in the structured element.

Another interesting application that utilizes polarization encoding is optical computing. Lohmann and Weigelt [1987] proposed a spatial filtering logic approach based on polarization. Using polarization logic instead of diffraction logic or scattering logic operations has several advantages. There is no loss of energy for both logic values 0 or 1, which is desirable for cascading elements, and polarization logic has a high space-bandwidth product. Figure 46 shows three different logical operations using a polarization-based spatial filtering logic method. Hashimoto, Kitayama and Mukohzaka [1989] proposed space-variant operations using optical parallel processor based on polarization encoding. A liquid-crystal spatial light modulator residing in the processor is used as an operational kernel. It enables a programmable space-variant operation to be performed on a real-time basis by spatially filtering the encoded light, pixel by pixel. Free-space optical



Fig. 46. Truth table for all sixteen binary logic operations and the corresponding filters in the second Fourier plane. The right column shows results of laboratory experiments, which were made visible by an analyzer (white = logical level 1, black = logical level 0). See also pages 134 and 135. Partly taken from fig. 4 in the work of Lohmann and Weigelt [1987].

interconnects offer low cross talk, high bandwidth, and parallel operation, and are therefore attractive for use in digital optical computers. Two particularly useful interconnect schemes are based on the perfect-shuffle transform and its inverse. Davidson, Friesem and Hasman [1992a] proposed realizing inverse perfect shuffle by use of space-variant polarization-state manipulation. Their arrangement for optical implementation of one-dimensional inverse perfect shuffle is shown in fig. 47. The input is coded with an interlaced polarizing mask. The odd pixels are covered with vertical polarizers whereas the even pixels are covered with horizontal polarizers. The input is illuminated with diffuse laser light that is derived from an argon laser ( $\lambda = 514.5$  nm). The holographic optical element is composed of two sub-holograms. The first sub-hologram is covered with a vertical polarizer so as to transmit light coming from only the odd pixels, and the second sub-hologram is covered with a horizontal polarizer so as to transmit light coming only from the even pixels. As this method for perfect shuffle uses only a single holographic optical element, it is simple, lightweight and compact.



Fig. 47. Optical arrangement for implementing a one-dimensional PS<sup>-1</sup> transform. (From Davidson, Friesem and Hasman [1992a].)

### 4.4. Space-variant polarization-dependent emissivity

Thermal emission from the bulk of a smooth, absorbing material is considered to be incoherent and unpolarized, thus, it is correlated to spontaneous emission. The surface properties of the absorbing material have a profound impact on its optical properties, and can lead to partially coherent and partially polarized radiation emission. Raether [1988] argued that in the case of materials with a dielectric constant that has a negative real part, surface waves provide the connection between the emission or absorption properties of the material and the surface properties. There are two kinds of materials, that support surface waves: conductive materials that support surface plasmon polaritons, and dielectric materials, that support surface phonon polaritons. Surface plasmon polaritons are due to an acoustic type of oscillation of the electron gas. Therefore, the surface electromagnetic waves are actually charge-density waves. The underlying microscopic origin of the surface phonon polariton is the mechanical vibration of the atoms or phonons (Marquier, Joulain, Mulet, Carminati, Greffet and Chen [2004]).

A surface polariton has a longer wavevector than the light-waves propagating along the surface with the same frequency. Therefore, they are called "nonradiative" surface polaritons. Their electromagnetic fields decay exponentially into space perpendicular to the surface and have their maximum value in the surface, as is characteristic of surface waves. In order to couple a propagating wave with the surface polariton, an additional prism or grating is needed. In this case, the

 $[4, \S 4]$ 

coupling is obtained for a certain frequency at a well-defined, propagating wave direction. When using a grating, the relationship between the emission angle  $\zeta$  and the wavelength  $\lambda$  is simply given by the usual grating law,

$$\frac{2\pi}{\lambda}\sin\zeta = k_{sp} + p\frac{2\pi}{\Lambda},\tag{4.6}$$

where *p* is an integer, *A* is the grating period, and  $k_{sp}$  is the wavevector of the surface wave. The connection between  $k_{sp}$ ,  $\lambda$  and the real part of the substrate's dielectric constant,  $\varepsilon'$ , is given by

$$k_{sp} = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon'}{1+\varepsilon'}}.$$
(4.7)

The coupling between the surface polaritons and the propagating wave can lead either to an increased resonant absorption or to directional emission. Surface polaritons can be coupled only with TM-polarized propagating waves, and as a result the absorption or emission is polarization dependent (see Setälä, Kaivola and Friberg [2002]).

Greffet, Carminati, Joulain, Mulet, Mainguy and Chen [2002] used surface wave theory to design and optimize a grating, ruled on a SiC substrate, that produced a strong peak of the emissivity around a wavelength  $\lambda = 11.36 \,\mu\text{m}$ . They measured the spectral reflection in various directions in order to obtain the emissivity using Kirchhoff's law  $\varepsilon = \alpha = 1 - R$ , where  $\varepsilon$ ,  $\alpha$  and R denote the emissivity, the absorption and the reflectivity, respectively. Their results are shown in fig. 48. The emission spectra of the thermal source are directionally dependent, as



Fig. 48. Emissivity of a SiC grating in TM-polarization. Line (a):  $\lambda = 11.04 \mu m$ ; line (b):  $\lambda = 11.36 \mu m$ ; line (c):  $\lambda = 11.86 \mu m$ . The emissivity was deduced from measurements of the specular reflectivity *R* using Kirchhoff's law. The data have been taken at ambient temperature using a Fourier-transform infrared (FTIR) spectrometer as a source and a detector mounted on a rotating arm. The angular acceptance of the spectrometer was reduced to a value lower than the angular width of the dip. The experimental data are indicated by circles; the lines show the theoretical results. (From Greffet, Carminati, Joulain, Mulet, Mainguy and Chen [2002].)

predicted by Wolf [1987]. They approximated the coherence length as  $\lambda/\zeta$  to be 60 $\lambda$ . Furthermore, Marquier, Joulain, Mulet, Carminati, Greffet and Chen [2004] showed that in certain frequencies the emission could be frequency-resonant and nondirectional.

Spatial variation of the emissivity can be obtained by using space-variant gratings embedded in a polar material (Dahan, Niv, Biener, Kleiner, Hasman [2005]). Accordingly, by spatially controlling the emissivity, we can generate spatially varying polarized fields. These can be used in various applications such as thermal polarization imaging, optical encryption, spatially modulated heat transfer, and the formation of high-efficiency thermal sources. We realized four space-variant elements with local groove orientations of  $\theta = \frac{1}{2}m\omega$ , where m is the polarization order number and  $\omega$  is the azimuthal angle of the polar coordinates. Such an element forms a spiral-like intensity and is appropriately called a spiral element. The elements formed were designed for polarization order numbers m = 1, 2, 3and 4. We optimized such a grating using RCWA to receive maximum emission at a wavelength of 9 µm on a fused silica substrate, which is a polar material. The grating period was 2  $\mu$ m with a fill factor of 0.3 and a depth of 0.8  $\mu$ m. It was fabricated using advanced photolithographic techniques. In order to reduce signal-to-noise ratio the realized element was heated to 80 °C. An image of the four elements, captured using a thermal camera with and without a polarizeranalyzer, is shown in fig. 49. Evidently, space-variant intensity modulation was obtained using the polarizer-analyzer due to space-variant polarization-dependent emissivity.



Fig. 49. Thermal image of four SiO<sub>2</sub> spiral elements. The elements are at uniform temperature of 80 °C hence the intensity is proportional to the emissivity. (a) With analyzer one can observe space-variant intensity. The polarization direction of the radiation from the dark sectors is perpendicular to the bright sectors. (b) Without analyzer, one observes uniform intensity.

#### § 5. Concluding remarks

Computer-generated holograms (Brown and Lohmann [1966], Lee [1974, 1978]) and diffractive optics had revolutionized the field of optics by allowing the formation of scalar fields with arbitrary phase structures. More recently, space-variant polarization manipulation using computer-generated polarization elements such as sub-wavelength gratings and liquid-crystal modulators has led to new approaches for obtaining complex fields.

In this review we have explored the nature of beams with space-variant polarization-state distributions. We began by discussing several possible methods for forming beams. These included computer-generated sub-wavelength metal or dielectric gratings, polarization interferometric methods, liquid-crystal devices and polarization-sensitive recording materials. The extensive discussion on subwavelength gratings included a theoretical background of sub-wavelength gratings along with theoretical analysis and experimental demonstration of spacevariant sub-wavelength metal and dielectric gratings. Several general design approaches for space-variant polarization optics were reviewed.

Space-variant polarization-state manipulations are necessarily accompanied by a geometrical phase – the Pancharatnam–Berry phase. We have demonstrated the generation of optical phase elements based on a space-domain Pancharatnam–Berry phase (Pancharatnam–Berry-phase optical elements, PBOEs). We then discussed the ability to utilize this phase to form sophisticated scalar as well as vectorial wavefronts. Following this, the effect of this geometrical phase on the propagation of a vectorial beam was explored. We believe that PBOEs will advance a variety of applications in modern optics and will lead to novel approaches in nano-optics as well.

We then moved on to review several applications involving space-variant polarization-state manipulation. These applications included near-field and farfield polarimetry, imaging polarimetry, spatial polarization scrambling, namely depolarizers, polarization encryption and polarization encoding. A preliminary study of space-variant polarization-dependent emissivity was presented.

Theoretical research involving space-variant polarization distribution is still in the primary stages and experimental demonstrations are somewhat limited. Nevertheless, this field has great potential to influence several other fields such as bioimaging and biosensing, optical tweezing and optical computing among others. To indicate the great interest in space-variant polarization manipulation, we can cite several preliminary works in these fields. Hielscher, Eick, Mourant, Shen, Freyer and Bigio [1997] reported on measuring the Mueller matrix of a cancerous and a noncancerous cell suspension. More recently, Galajda and Ormos [2003] described the effect of using polarized light on the trapping of nonspherical beads. There are numerous other studies dealing with the influence of polarized light on light–matter interactions. It would seem that there are countless possible areas to explore dealing with space-variant polarized beams. One such area is that of optical computing, and we cited two studies that exploited the properties of polarization – the logical operation suggested by Lohmann and Weigelt [1987] and the interconnect proposed by Davidson, Friesem and Hasman [1992a].

This review has focused mainly on polarized, coherent and monochromatic light beams. Further experimental and theoretical investigations should be conducted on partially polarized, partially coherent, polychromatic light beams with space-variant polarization-state distribution (Gori, Santarsiero, Borghi and Piquero [2000]). These investigations should also include nonparaxial beams. Finally, more comprehensive research should be conducted in the field of polarization thermal emissivity, with the emphasis on the interaction between space-variant polarized coherent emission and the excitation of surface phonon or plasmon polariton resonance.

### References

- Allen, L., Padgett, M.J., Babiker, M., 1999, in: Wolf, E. (Ed.), Progress in Optics, vol. 39, North-Holland, Amsterdam, pp. 291–370.
- Anderson, D.G.M., Barakat, R., 1994, J. Opt. Soc. Am. A 11, 2305.
- Arrizón, V., Tepichin, E., Ortiz-Gutierrez, M., Lohmann, A.W., 1996, Opt. Commun. 127, 171.
- Astilean, S., Lalanne, P., Palamaru, M., 2000, Opt. Commun. 175, 265.
- d'Auria, L., Huignard, J.P., Roy, A.M., Spitz, E., 1972, Opt. Commun. 5, 232.
- Azzam, R.M.A., 1987, Opt. Lett. 12, 555.
- Baba, N., Murakami, N., Ishigaki, T., 2001, Opt. Lett. 26, 1167.
- Barakat, R., 1987, Appl. Opt. 26, 3790.
- Beijersbergen, M.W., Coerwinkel, R.P.C., Kristensen, M., Woerdman, J.P., 1994, Opt. Commun. 112, 321.
- Berry, M.V., 1984, Proc. R. Soc. London Ser. A 392, 45.
- Berry, M.V., 1987, J. Mod. Opt. 34, 1401.
- Bhandari, R., 1995, Phys. Lett. A 204, 188.
- Bhandari, R., 1997, Phys. Rep. 281, 1.
- Biener, G., Niv, A., Kleiner, V., Hasman, E., 2002, Opt. Lett. 27, 1875.
- Biener, G., Niv, A., Kleiner, V., Hasman, E., 2003a, Opt. Lett. 28, 1400.
- Biener, G., Niv, A., Kleiner, V., Hasman, E., 2003b, J. Opt. Soc. Am. A 20, 1940.
- Biener, G., Niv, A., Kleiner, V., Hasman, E., 2005, Opt. Lett. 30, 1096.
- Billings, B.H., 1951, J. Opt. Soc. Am. 41, 966.
- Bird, G.R., Parrish Jr., M., 1960, J. Opt. Soc. Am. 50, 886.
- Bomzon, Z., Biener, G., Kleiner, V., Hasman, E., 2002a, Opt. Lett. 27, 188.
- Bomzon, Z., Biener, G., Kleiner, V., Hasman, E., 2002b, Opt. Lett. 27, 285.
- Bomzon, Z., Biener, G., Kleiner, V., Hasman, E., 2002c, Opt. Lett. 27, 1141.
- Bomzon, Z., Kleiner, V., Hasman, E., 2001a, Opt. Lett. 26, 33.

- Bomzon, Z., Kleiner, V., Hasman, E., 2001b, Opt. Commun. 192, 169.
- Bomzon, Z., Kleiner, V., Hasman, E., 2001c, Appl. Phys. Lett. 79, 1587.
- Bomzon, Z., Kleiner, V., Hasman, E., 2001d, Opt. Lett. 26, 1424.
- Bomzon, Z., Niv, A., Biener, G., Kleiner, V., Hasman, E., 2002a, Appl. Phys. Lett. 80, 3685.
- Bomzon, Z., Niv, A., Biener, G., Kleiner, V., Hasman, E., 2002b, Appl. Opt. 41, 5218.
- Born, M., Wolf, E., 1999, Principles of Optics, Cambridge University Press, Cambridge.
- Bouchitte, G., Petit, R., 1985, Electromagnetics 5, 17.
- Bowden, M.J., 1994, in: Thompson, L.F., Wilson, C.G., Bowden, M.J. (Eds.), Introduction to Microlithography, second ed., American Chemical Society, Washington, DC, pp. 19–136.
- Brosseau, C., 1985, Optik 72, 38.
- Brosseau, C., 1998, Fundamentals of Polarized Light, Wiley, New York.
- Brown, B.R., Lohmann, A.W., 1966, Appl. Opt. 5, 967.
- Brundrett, D.L., Gaylord, T.K., Glytsis, E.N., 1998, Appl. Opt. 37, 2534.
- Brundrett, D.L., Glytsis, E.N., Gaylord, T.K., 1996, Appl. Opt. 35, 6195.
- Challener, W.A., 1996, J. Opt. Soc. Am. A 13, 1859.
- Chou, P.C., Fini, J.M., Haus, H.A., 2001, IEEE Photonics Technol. Lett. 13, 568.
- Cincotti, G., 2003, IEEE J. Quantum Electron. 39, 1645.
- Ciuchi, F., Mazzulla, A., Cipparrone, G., 2002, J. Opt. Soc. Am. B 19, 2531.
- Collett, E., 1993, Polarized Light, Marcel Dekker, New York.
- Collett, E., 2003, Polarized Light in Fiber Optics, The PolaWave Group, Lincroft, NJ.
- Dahan, N., Niv, A., Biener, G., Kleiner, V., Hasman, E., 2005, Appl. Phys. Lett. 86, 191102.
- Dammann, H., 1970, Optik 31, 95.
- Davidson, N., Bokor, N., 2004, Opt. Lett. 29, 1318.
- Davidson, N., Friesem, A.A., Hasman, E., 1992a, Appl. Opt. 31, 1810.
- Davidson, N., Friesem, A.A., Hasman, E., 1992b, Opt. Commun. 88, 326.
- Davis, J.A., Adachi, J., Fernández-Pousa, C.R., Moreno, I., 2001, Opt. Lett. 26, 587.
- Davis, J.A., McNamara, D.E., Cottrell, D.M., Sonehara, T., 2000, Appl. Opt. 39, 1549.
- Debye, P., 1909, Ann. Phys. 30, 755.
- Deguzman, P.C., Nordin, G.P., 2001, Appl. Opt. 40, 5731.
- Dennis, M.R., 2002, Opt. Commun. 213, 201.
- Dorn, R., Quabis, S., Leuchs, G., 2003, Phys. Rev. Lett. 91, 233901.
- Doumuki, T., Tamada, H., 1997, Appl. Phys. Lett. 71, 686.
- Durnin, J., Miceli Jr., J.J., Eberly, J.H., 1987, Phys. Rev. Lett. 58, 1499.
- Enger, R.C., Case, S.K., 1983, Appl. Opt. 22, 3220.
- Eriksen, R.L., Mogensen, P.C., Glückstad, J., 2001, Opt. Commun. 187, 325.
- Fernández-Pousa, C.R., Moreno, I., Davis, J.A., Adachi, J., 2001, Opt. Lett. 26, 1651.
- Ferrari, J.A., Garbusi, E., Frins, E.M., 2004, Opt. Lett. 29, 1138.
- Freund, I., Mokhun, A.I., Soskin, M.S., Angelsky, O.V., Mokhun, I.I., 2002, Opt. Lett. 27, 545.
- Frins, E.M., Ferrari, J.A., Dubra, A., Perciante, D., 2000, Opt. Lett. 25, 284.
- Galajda, P., Ormos, P., 2003, Opt. Exp. 11, 446.
- Garcés-Chávez, V., McGloin, D., Melville, H., Sibbett, W., Dholakia, K., 2002, Nature 419, 145.
- Glytsis, E.N., Gaylord, T.K., 1992, Appl. Opt. 31, 4459.
- Goodman, J.W., 1996, Introduction to Fourier Optics, McGraw-Hill, Singapore.
- Gori, F., 1999, Opt. Lett. 24, 584.
- Gori, F., 2001, J. Opt. Soc. Am. A 18, 1612.
- Gori, F., Santarsiero, M., Borghi, R., Piquero, G., 2000, Opt. Lett. 25, 1291.
- Gori, F., Santarsiero, M., Vicalvi, S., Borghi, R., Guattari, G., 1998, Pure Appl. Opt. 7, 941.
- Grann, E.B., Moharam, M.G., Pommet, D.A., 1994, J. Opt. Soc. Am. A 11, 2695.
- Greffet, J.J., Carminati, R., Joulain, K., Mulet, J.P., Mainguy, S., Chen, Y., 2002, Nature **416**, 61. Guizal, B., Felbacq, D., 1999, Opt. Commun. **165**, 1.
- Guo, J., Brady, D., 2000, Appl. Opt. 39, 1486.
- Hashimoto, M., Kitayama, K.I., Mukohzaka, N., 1989, Opt. Lett. 14, 384.
- Hasman, E., Biener, G., Kleiner, V., Niv, A., 2003, Opt. Photon. News 14, 34.
- Hasman, E., Bomzon, Z., Niv, A., Biener, G., Kleiner, V., 2002, Opt. Commun. 209, 45.
- Hasman, E., Davidson, N., Friesem, A.A., 1991, Opt. Lett. 16, 423.
- Hasman, E., Kleiner, V., Biener, G., Niv, A., 2003, Appl. Phys. Lett. 82, 328.
- Heismann, F., Tokuda, K.L., 1995, Opt. Lett. 20, 1008.
- Hertz, H., 1893, Electric Waves, Macmillan and Company, London.
- Hielscher, A.H., Eick, A.A., Mourant, J.R., Shen, D., Freyer, J.P., Bigio, I.J., 1997, Opt. Exp. 1, 441.
- Holme, N.C.R., Ramanujam, P.S., Hvilsted, S., 1996, Opt. Lett. 21, 902.
- Honkanen, M., Kettunen, V., Kuittinen, M., Lautanen, J., Turunen, J., Schnabel, B., Wyrowski, F., 1999, Appl. Phys. B 68, 81.
- Hopkins, H.H., 1943, Proc. Phys. Soc. London 55, 116.
- James, D.F.V., 1994, J. Opt. Soc. Am. A 11, 1641.
- Javidi, B., Nomura, T., 2000, Opt. Eng. 39, 2439.
- Jellison Jr, G.E., 1987, Opt. Lett. 12, 766.
- Jiang, J., Nordin, G.P., 2000, Opt. Exp. 7, 237.
- Jones, R.C., 1948, J. Opt. Soc. Am. 38, 671.
- Jordan, R.H., Hall, D.G., 1994, Opt. Lett. 19, 427.
- Kakichashvili, S.D., 1972, Opt. Spectrosc. 33, 171.
- Kawano, K., Ishii, T., Minabe, J., Niitsu, T., Nishikata, Y., Baba, K., 1999, Opt. Lett. 24, 1269.
- Kersey, A.D., Marrone, M.J., Dandridge, A., 1990, J. Lightwave Technol. 8, 838.
- Kung, H.L., Bhatnagar, A., Miller, D.A.B., 2001, Opt. Lett. 26, 1645.
- Kwiat, P.G., Chiao, R.Y., 1991, Phys. Rev. Lett. 66, 588.
- Lalanne, P., Morris, G.M., 1996, J. Opt. Soc. Am. A 13, 779.
- Lee, J., Koh, J., Collins, R.W., 2000, Opt. Lett. 25, 1573.
- Lee, W.H., 1974, Appl. Opt. 13, 1677.
- Lee, W.H., 1978, in: Wolf, E. (Ed.), Progress in Optics, vol. 16, North-Holland, Amsterdam, p. 119.
- Liu, J., Gu, B.Y., Yang, G.Z., 1999, Appl. Opt. 38, 6887.
- Liu, Y., Cline, D., He, P., 1999, Nucl. Instrum. Meth. Phys. Res. A 424, 296.
- Lopez, A.G., Craighead, H.G., 1998, Opt. Lett. 23, 1627.
- Lohmann, A.W., Weigelt, J., 1987, Appl. Opt. 26, 131.
- Lu, S.Y., Chipman, R.A., 1998, Opt. Commun. 146, 11.
- Lyot, B.F., 1928, Ann. Observatoire Astron. Phys. Paris.
- MacDonald, M.P., Paterson, L., Volke-Sepulveda, K., Arlt, J., Sibbett, W., Dholakia, K., 2002, Science 296, 1101.
- Macleod, H.A., 1989, Thin-Film Optical Filters, McGraw-Hill, Bristol.
- Mair, A., Vaziri, A., Weihs, G., Zeilinger, A., 2001, Nature 412, 313.
- Mait, J.N., Prather, D.W., Mirotznik, M.S., 1999, J. Opt. Soc. Am. A 16, 1157.
- Mandel, L., Wolf, E., 1995, Optical Coherence and Quantum Optics, Cambridge University Press, Cambridge.
- Marquier, F., Joulain, K., Mulet, J.P., Carminati, R., Greffet, J.J., Chen, Y., 2004, Phys. Rev. B 69, 155412.
- Matoba, O., Javidi, B., 2004, Appl. Opt. 43, 2915.
- Mazurczyk, V.J., Zyskind, J.L., 1994, IEEE Photonics Technol. Lett. 6, 616.
- McGuire Jr., J.P., Chipman, R.A., 1990, Opt. Eng. 29, 1478.
- Mirotznik, M.S., Prather, D.W., Mait, J.N., Beck, W.A., Shi, S., Gao, X., 2000, Appl. Opt. 39, 2871.
- Mishra, S.R., 1991, Opt. Commun. 85, 159.
- Mogensen, P.C., Glückstad, J., 2000, Opt. Commun. 173, 177.
- Moharam, M.G., Gaylord, T.K., 1986, J. Opt. Soc. Am. A 3, 1780.

- Mushiake, Y., Matsumura, K., Nakajima, N., 1972, Proc. IEEE 60, 1107.
- Neil, M.A.A., Massoumian, F., Juškaitis, R., Wilson, T., 2002, Opt. Lett. 27, 1929.
- Nesterov, A.V., Niziev, V.G., Yakunin, V.P., 1999, J. Phys. D 32, 2871.
- Niv, A., Biener, G., Kleiner, V., Hasman, E., 2003, Opt. Lett. 28, 510.
- Niv, A., Biener, G., Kleiner, V., Hasman, E., 2004, Opt. Lett. 29, 238.
- Niziev, V.G., Nesterov, A.V., 1999, J. Phys. D 32, 1455.
- Nordin, G.P., Meier, J.T., Deguzman, P.C., Jones, M.W., 1999, J. Opt. Soc. Am. A 16, 1168.
- Nye, J.F., 1983, Proc. R. Soc. London Ser. A 387, 105.
- Nye, J.F., 1999, Natural Focusing and Fine Structure of Light, Institute of Physics Publishing, Bristol.
- Oka, K., Kaneko, T., 2003, Opt. Exp. 11, 1510.
- Oka, K., Kato, T., 1999, Opt. Lett. 24, 1475.
- Oron, R., Blit, S., Davidson, N., Friesem, A.A., Bomzon, Z., Hasman, E., 2000, Appl. Phys. Lett. 77, 3322.
- Oron, R., Davidson, N., Friesem, A.A., Hasman, E., 2001, in: Wolf, E. (Ed.), Progress in Optics, vol. 42, Elsevier, Amsterdam, p. 325.
- Pääkkönen, P., Tervo, J., Vahimaa, P., Turunen, J., Gori, F., 2002, Opt. Exp. 10, 949.
- Pancharatnam, S., 1956, Proc. Ind. Acad. Sci. 44, 247.
- Pancharatnam, S., 1975, Collected Works, Oxford University Press, Oxford.
- Paterson, L., MacDonald, M.P., Arlt, J., Sibbett, W., Bryant, P.E., Dholakia, K., 2001, Science 292, 912.
- Pohl, D., 1972, Appl. Phys. Lett. 20, 266.
- Puscasu, I., Spencer, D., Boreman, G.D., 2000, Appl. Opt. 39, 1570.
- Quabis, S., Dorn, R., Eberler, M., Glöckl, O., Leuchs, G., 2000, Opt. Commun. 179, 1.
- Quabis, S., Dorn, R., Eberler, M., Glöckl, O., Leuchs, G., 2001, Appl. Phys. B 72, 109.
- Raab, R.E., 1982, Opt. Acta 29, 1243.
- Rabal, H.J., Furlan, W.D., Sicre, E.E., 1986, Opt. Commun. 57, 81.
- Raether, H., 1988, Surface Plasmons, Springer-Verlag, Berlin.
- Refregier, P., Javidi, B., 1995, Opt. Lett. 20, 767.
- Richards, B., Wolf, E., 1959, Proc. R. Soc. London Ser. A 253, 358.
- Rytov, S.M., 1956, Sov. Phys. JETP 2, 466.
- Sacks, Z.S., Rozas, D., Swartzlander Jr., G.A., 1998, J. Opt. Soc. Am. B 15, 2226.
- Sankaran, V., Everett, M.J., Maitland, D.J., Walsh Jr., J.T., 1999, Opt. Lett. 24, 1044.
- Schnabel, B., Kley, E.B., Wyrowski, F., 1999, Opt. Eng. 38, 220.
- Seshadri, S.R., 1999, J. Opt. Soc. Am. A 16, 1373.
- Setälä, T., Kaivola, M., Friberg, A.T., 2002, Phys. Rev. Lett. 88, 123902.
- Shapere, A., Wilczek, F., 1989, Geometric Phases in Physics, World Scientific, Singapore.
- Sheppard, C.J.R., Wilson, T., 1982, Proc. R. Soc. London Ser. A 379, 145.
- Siegel, C., Loewenthal, F., Balmer, J.E., 2001, Opt. Commun. 194, 265.
- Simon, R., Kimble, H.J., Sudarshan, E.C.G., 1988, Phys. Rev. Lett. 61, 19.
- Soskin, M.S., Vasnetsov, M.V., 2001, in: Wolf, E. (Ed.), Progress in Optics, vol. 42, Elsevier, Amsterdam, pp. 219–276, chapter 4.
- Stalder, M., Schadt, M., 1996, Opt. Lett. 21, 1948.
- Stokes, G.G., 1852, Trans. Camb. Philos. Soc. 9, 399. Reprinted in: Mathematical and Physical Papers, vol. 3, Cambridge University Press, London, 1901, p. 233.
- Talbot, F., 1836, Philos. Mag. 9, 401.
- Tan, X., Matoba, O., Okada-Shudo, Y., Ide, M., Shimura, T., Kuroda, K., 2001, Appl. Opt. 40, 2310.
- Tervo, J., Kettunen, V., Honkanen, M., Turunen, J., 2003, J. Opt. Soc. Am. A 20, 282.
- Tervo, J., Turunen, J., 2000, Opt. Lett. 25, 785.
- Tervo, J., Turunen, J., 2001, Opt. Commun. 192, 13.
- Tidwell, S.C., Ford, D.H., Kimura, W.D., 1990, Appl. Opt. 29, 2234.

- Tidwell, S.C., Kim, G.H., Kimura, W.D., 1993, Appl. Opt. 32, 5222.
- Todorov, T., Nikolova, L., Tomova, N., 1984, Appl. Opt. 23, 4309.
- Tovar, A.A., 1998, J. Opt. Soc. Am. A 15, 2705.
- Towghi, N., Javidi, B., Luo, Z., 1999, J. Opt. Soc. Am. A 16, 1915.
- Unnikrishnan, G., Joseph, J., Singh, K., 1998, Appl. Opt. 37, 8181.
- Unnikrishnan, G., Pohit, M., Singh, K., 2000, Opt. Commun. 185, 25.
- Van Delden, J.S., 2003, Opt. Lett. 28, 1173.
- Warren, M.E., Smith, R.E., Vawter, G.A., Wendt, J.R., 1995, Opt. Lett. 20, 1441.
- Wen, B., Petschek, R.G., Rosenblatt, C., 2002, Appl. Opt. 41, 1246.
- Wolf, E., 1954, Nuovo Cimento 12, 884.
- Wolf, E., 1959, Proc. R. Soc. London Ser. A 253, 349.
- Wolf, E., 1987, Nature 326, 363.
- Wrage, M., Glas, P., Fischer, D., Leitner, M., Vysotsky, D.V., Napartovich, A.P., 2000, Opt. Lett. 25, 1436.
- Wynne, J.J., 1974, IEEE J. Quantum Electron. 10, 125.
- Xu, F., Tyan, R.C., Sun, P.C., Fainman, Y., Cheng, C.C., Scherer, A., 1996, Opt. Lett. 21, 1513.
- Yeh, P., 1979, J. Opt. Soc. Am. 69, 742.
- Zeitner, U.D., Schnabel, B., Kley, E.B., Wyrowski, F., 1999, Appl. Opt. 38, 2177.
- Zhan, Q., Leger, J.R., 2002, Opt. Commun. 213, 241.