Supplementary Materials for
Photonic topological spin Hall effect mediated by vortex pairs

Bo Wang (王波)†, Elhanan Maguid†, Kexiu Rong (容科秀), Michael Yannai, Vladimir Kleiner
and Erez Hasman*
† These authors contributed equally to this work
Micro and Nanooptics Laboratory, Faculty of Mechanical Engineering and Russell Berrie
Nanotechnology Institute, Technion – Israel Institute of Technology, Haifa 3200003, Israel.
Corresponding Author: mehasman@technion.ac.il;

The supplementary materials consist of 7 sections in support of the main body of the text. The contents of the sections are listed below:

Section S1: Metasurface design and fabrication

Section S2: Wigner distribution

Section S3: Interference patterns of VPs

Section S4: Measurement of the correlation parameter \( \alpha \)

Section S5: Weak measurement of PSHE

Section S6: Photonic random Rashba effect and correlation functions

Section S7: VP model

Figures S1-S10
**Section S1: Metasurface design and fabrication**

Figure 2(c) depicts the SEM images of resultant Si-based GPMs with different defect concentrations. Each building block is composed of several nanorods, filling an area of $600\times600$ nm$^2$. The nanorods are of 80 nm width and 300 nm depth, arranged 200 nm apart from each other. The metasurface diameter is $D = 200$ µm.

The poly-Si thin film with a thickness of 300 nm was grown at a temperature of 590°C on a SiO$_2$ (fused silica) substrate. The photoresist CSAR 6200.09 (Allresist GmbH) with a thickness of 190 nm was deposited on the poly-Si film and baked at 150°C for 1 minute. To transfer the pattern onto the poly-Si film, a photoresist mask was made using a Raith EBPG E-beam lithography system, then baked at 130 °C for 1 minute and etched using deep reactive-ion etching by F-ICP Plasma Therm system for 30 seconds.

**Section S2: Wigner distribution**

The number of topological defects that emerge in the near-field of the metasurface is analyzed in terms of Wigner position-momentum $(x,k)$ distribution in phase space [28]. The Wigner function of a GPM is represented by

$$W^{(\zeta)}(x,k) = \int \psi^{(\zeta)}_\sigma(x + \xi) \{\psi^{(\zeta)}_\sigma(x - \xi)\}^\dagger \exp(2i\xi \cdot k) d\xi,$$

where $\psi^{(\zeta)}_\sigma(x) = \exp(i\phi^{(\zeta)}_\sigma(x))$ is the phase function corresponding to a metasurface of $\zeta$ defect concentration for $\sigma = \pm 1$ illumination. The Wigner phase-space distribution entropy is analyzed in terms of the mutual information of the GPMs

$$MI^{(\zeta)} = \int W^{(\zeta)}(x,k) \log_2 \left[ \frac{W^{(\zeta)}(x,k)}{\rho^{(\zeta)}_\sigma(x)\pi^{(\zeta)}_\sigma(k)} \right] dkd\mathbf{x},$$

where $\rho^{(\zeta)}_\sigma(x) = \int W^{(\zeta)}_\sigma(x,k)dk$ and $\pi^{(\zeta)}_\sigma(k) = \int W^{(\zeta)}_\sigma(x,k)dx$ are the marginal density functions in position and momentum space, respectively.

**Section S3: Interference patterns of VPs**

The demonstration of bound VPs, two closely located topological defects with topological charges of $l = \pm 1$, was obtained by calculating the propagation of the spin-flipped field over a distance of a wavelength above the metasurface. The calculation is performed by assuming that each point in the metasurface evolves according to Huygens’ principle.
~ \exp(-i2\sigma \theta(x,y)) \cdot \exp(ik_0r_0/r_0), \text{ where } k_0 \text{ is the wavenumber and } r_0 \text{ is the distance between the calculation area and the point in the metasurface. Figure S1 (a) depicts the phase distribution emerging from a GPM with low defect concentration for a \sigma_+ illumination. Figure S1 (b) shows the interference pattern between the emerging field and a plane wave. The bound VPs are manifested as fork pairs, indicating that the VP consists of topological charges } l = \pm 1.

In order to observe such interference patterns we used the setup depicted in Fig. S2. To this end, a circularly polarized beam is split into two branches, one of which passes through the GPM obtaining a spin-dependent phase accumulation of the flipped spin state with respect to the incident beam (the spin-flipped component was selected by a circular polarization analyzer, which was not shown in the setup sketch). The other branch (reference beam) passes through a HWP to flip the incident spin state. The experimental interference patterns and the corresponding near-field intensity distributions for different defect concentrations are shown in Fig. S3. The near-field intensity distributions are obtained by blocking the reference beam. We observed intensity singularities and fork pairs at the same locations in the near field for low defect concentrations. The number of VPs increases along with the defect concentration up to a point where we can no longer identify the pairs.

**Section S4: Measurement of the correlation parameter \( \alpha \)**

The first-order correlation function is given by \( c(|\mathbf{r} - \mathbf{r}'|) = \langle \psi(\mathbf{r})^* \psi(\mathbf{r}') \rangle \), where \( \psi \) is the complex electric field and \( \mathbf{r}, \mathbf{r}' \) are coordinate vectors defined as \( \mathbf{r} = (x, y) \) and \( \mathbf{r}' = (x', y') \). The correlation parameter \( \alpha \) which describes the decay of the correlation \( C^2(R) \sim R^{-2\alpha} \), indicating the length-scale ordering of the system. Here

\[
C^2(R) = (1/R) \int_0^R dR' \left[ c(R') \right]^2;
\]

\( R' = |\mathbf{r} - \mathbf{r}'| \) is a variable distance, the maximum of which is the metasurface diameter \( D \). The complex near field \( \psi(\mathbf{r}) \) can not be observed directly, but it can be obtained by the experimental method of interference.
The interference pattern in Fig. S4 can be expressed as \( I(r) = |\psi(r) + \chi e^{ik \cdot r}|^2 \), where \( \psi(r) \) is the spin-flipped near field emerging from the metasurface and \( \chi e^{ik \cdot r} \) is the reference plane wave with a tilted wave vector \( k_r \). Applying Fourier analysis to \( I(r) \), and taking only the first diffraction order centered around \( k = k_r \) (lower right panel in Fig. S4), the complex coherence was achieved. Applying an inverse Fourier analysis to this specific part of complex coherence, we obtained a field \( \psi_e(r) \). We demonstrated that \( \psi_e(r) = \psi(r) \), as depicted in Fig. S4. At last, the experimental correlation parameter \( \alpha \) was obtained by calculating the correlations \( c(r - r') \) and \( C^2(R) \) of the complex field \( \psi_e(r) \).

Section S5: Weak measurement of PSHE

The experimental observation of the PSHE was obtained by using weak measurement to enable amplification of the sub-diffraction-limited beam shift [12]. Amplification of a weak value of the measured observable \( \hat{A} \) is achieved by applying pre-selection and post-selection polarization projections, i.e. \( |\psi_{\text{pre}}\rangle \) and \( |\psi_{\text{post}}\rangle \), resulting in an amplified value \( A_w = \langle \psi_{\text{pre}} | \hat{A} | \psi_{\text{post}} \rangle / \langle \psi_{\text{pre}} | \psi_{\text{post}} \rangle \).

We utilized the weak measurement technique to measure a sub-diffraction-limited spin-dependent deflection \( \delta \) where the pre- and post-selection states are the polarization states of light. Therefore, we used the Jones calculus to perform the polarization analysis of the optical fields in the system consisting of a GPM, polarizers (Pol) and quarter-wave plates (QWP). Using the formalism for a linear polarization basis, a polarizer rotated by an angle \( \alpha \) is described by the matrix

\[
J_{\text{Pol}}(\alpha) = \begin{pmatrix}
\cos^2(\alpha) & \cos(\alpha)\sin(\alpha) \\
\cos(\alpha)\sin(\alpha) & \sin^2(\alpha)
\end{pmatrix},
\]

and a QWP rotated at an angle \( \beta \) by

\[
J_{\text{QWP}}(\beta) = \begin{pmatrix}
\cos^2(\beta) + i\sin^2(\beta) & (1-i)\cos(\beta)\sin(\beta) \\
(1-i)\cos(\beta)\sin(\beta) & \sin^2(\beta) + i\cos^2(\beta)
\end{pmatrix}.
\]

A GPM can be described as a spin-dependent matrix operator \( J_{\text{GPM}}(\sigma, k) \) for manipulation of the incident light field in momentum space. In the following, we use one-dimensional momentum
space to simplify the calculation, where \( A(k) = \exp \left( -\frac{|k|^2}{2k_w^2} \right) \) is a Gaussian amplitude profile and \( k_w \) is the diffraction-limited spot in momentum space. The sub-diffraction-limited spin-dependent deflection induced by the GPM can be written in a first-order approximation as

\[
J_{\text{GPM}}(\sigma,k)A(k)|\sigma_+\rangle = A(k)\left(1 - \frac{k\delta}{k_w^2}\right)|\sigma_+\rangle
\]

\[
J_{\text{GPM}}(\sigma,k)A(k)|\sigma_-\rangle = A(k)\left(1 + \frac{k\delta}{k_w^2}\right)|\sigma_-\rangle,
\]

where \(|\sigma_+\rangle\) and \(|\sigma_-\rangle\) are the spin states.

The output field obtained from the experiment (schematic in Fig. S5) can be described by

\[
E_{\text{out}}(k,\alpha_1,\beta_1,\alpha_2,\beta_2) = J_{\text{Pol}}(\alpha_2)J_{\text{QWP}}(\beta_2)J_{\text{GPM}}(\sigma,k)J_{\text{QWP}}(\beta_1)J_{\text{Pol}}(\alpha_1)A(k)E_{\text{in}}.\]

Here, \( E_{\text{in}} \) is the Jones vector of the incident field, and the subscripts 1 and 2 denote pre-selection and post-selection states, respectively. By setting the pre-selection state as \( \alpha_1 = \beta_1 = \pi/2 \), we obtain

\[
J_{\text{GPM}}(\sigma,k)J_{\text{QWP}}(\pi/2)J_{\text{Pol}}(\pi/2)A(k)E_{\text{in}} = \left[\frac{k\delta i}{k_w^2} \right]A(k).
\]

By setting the post-selection state as \( \alpha_2 = 0 \), the output field \( E_{\text{out}}(k,\beta_2) \) is

\[
E_{\text{out}}(k,0,\beta_2,\pi/2,\pi/2) = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right]\left[\begin{array}{cc} \cos^2(\beta_2) + i\sin^2(\beta_2) & (1-i)\cos(\beta_2)\sin(\beta_2) \\ (1-i)\cos(\beta_2)\sin(\beta_2) & \sin^2(\beta_2) + i\cos^2(\beta_2) \end{array}\right]\left[\begin{array}{c} k\delta i \\ k_w^2 \end{array}\right]A(k).
\]

For \( \beta_2 \ll 1 \),

\[
E_{\text{out}}(k,\beta_2) \approx \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]\left[\begin{array}{c} 1 \\ (1-i)\beta_2 \end{array}\right]\left[\begin{array}{c} k\delta i \\ k_w^2 \end{array}\right]A(k) = \left[\begin{array}{c} k\delta \frac{\beta_2}{k_w^2} + \beta_2 \end{array}\right]A(k).
\]

Therefore, the corresponding intensity is \( I_{\text{out}}(k,\beta_2) = |E_{\text{out}}(k,\beta_2)|^2 \). The measured gravity center of the post-selected beam is defined as \( \delta_M(\beta_2) = \int kI_{\text{out}}(k,\beta_2)dk / \int I_{\text{out}}(k,\beta_2)dk \). The spin-dependent deflection \( \delta \) can be evaluated by the gradient of the \( \delta_M(\Delta \theta) \) curve (Fig. S6). We can
conclude from Fig.S6 that the smaller the $\delta$, the narrower the regime will be. The maximum $\delta_m$ we can achieve depends on the diffraction limit spot in the momentum space.

Note that in the case of $\beta_2 >> \delta / k_w$, we get $\delta_m(\beta_2) = -\delta / \beta_2$, where the amplification factor is $1 / \beta_2$[12]. When the PSHE is induced by a GPM of moderate efficiency, the background noise of the spin-maintained component increases along with the angle $\beta_2$. Here $\beta_2 = \Delta \theta$ is set for an extremely small regime to avoid the noise (Fig. S6, inset).

The experimental $\delta_m$ as a function of the QWP angles (Fig. S7) was obtained from the momentum space intensity measurements at several defect concentrations (Fig. S8). The centroid of the post-selected beam varies with the QWP angle, manifested by the intensity balancing between the two lobes. The amplified PSHE $I(-\Delta \theta) - I(\Delta \theta)$, indicates the orientation of the spin-dependent deflection. The data of each graph are collected with a step of 0.05 degrees using an electric controlled rotator in a range of $\sim 3$ degrees.

Section S6: Photonic random Rashba effect and correlation functions

The experimental observation of numerous spin-split modes for a GPM with defect concentration $\xi = 1$ (Fig. S9 (a) and (b)) was analyzed using a correlation function

$$\langle I_{\sigma_+}(k)I_{\sigma_-}(k-\Delta k)\rangle = \frac{1}{N} \sum_k \left(I_{\sigma_+}(k) - \bar{I}_{\sigma_+}(k)\right)\left(I_{\sigma_-}(k-\Delta k) - \bar{I}_{\sigma_-}(k-\Delta k)\right),$$

where $I_{\sigma_+}(k)$ and $I_{\sigma_-}(k)$ are the spin up and spin down momentum space intensities, and $\bar{I}_{\sigma_+}, \bar{I}_{\sigma_-}$ are their mean values, respectively. Here, $\Delta k$ denotes a displacement in the momentum space and

$$N = \sqrt{\sum_k \left(I_{\sigma_+}(k) - \bar{I}_{\sigma_+}(k)\right)^2 \sum_k \left(I_{\sigma_-}(k-\Delta k) - \bar{I}_{\sigma_-}(k-\Delta k)\right)^2}$$

is a normalization factor.

For $\langle I_{\sigma_+}(k)I_{\sigma_-}(-k+\Delta k)\rangle$, a strong correlation was found (Fig. S9 (c) and (d)); however, for $\langle I_{\sigma_+}(k)I_{\sigma_-}(k+\Delta k)\rangle$, no correlation was found (Fig. S9 (e) and (f)).

Section S7: VP model

Using the Huygens’ principle, we calculated the near field emerging from a GPM with low defects, manifested by bound VPs. Here we concentrate on a single VP phase profile – two closely located phase singularities with spin-dependent topological charge +1 and -1 (Fig.
S10(a)). This phase distribution can be approximated to \( \phi_{\text{VP}} = \sigma \left[ \phi(r-r_1) - \phi(r-r_2) \right] \), where \( \phi \) is the azimuth angle of the in-plane vector \( r-r_1 \) and \( r-r_2 \) (Fig. S10 (b)). This model presents a spin-dependent VP as two opposite topological charges \( \pm 1 \) located at \( r_1 \) and \( r_2 \), respectively. Figure S10 demonstrates the similarity between the calculated near field emerging from the GPM and the VP model. This model facilitates the study of the optical system’s dynamics as the number of VPs changes, while neglecting the weak phase fluctuations. Inserting multiple VPs in the same plane was achieved by adding the total phase of VPs from different locations and modifying the final phase to the range of \( [-\pi \pi] \).

**Fig. S1.** Calculated near-field phase distribution and interference pattern for a GPM with a low defect concentration. The phase distribution in (a) is calculated by the Huygens’ principle for a \( \sigma_+ \) illumination. The interference pattern in (b) is the superposition of the field in (a) and a plane wave. The red and blue circles mark fork pairs with opposite topological charges \( \pm 1 \).
Fig. S2. Experiment setup for interference patterns measurement. HWP: half wave plate, BS: beam splitter, Pol: polarizer, QWP: quarter wave plate.
Fig. S3. Measured interference patterns and near-field intensity distributions for GPMs with different defect concentrations. The red circles and the insets emphasize the singularities as VPs and their fork pair patterns.
**Fig. S4.** Schematic of the process for extracting the near-field phase distributions from the measured interference patterns. The processes show that a phase distribution is retrievable from an interference pattern in the experiment.

**Fig. S5.** Experiment setup for the weak measurement. The black arrows indicate the polarization directions and the blue arrows indicate the directions of fast axis.
Fig. S6. Calculated $\delta_{m}$ as a function of the QWP angle in post-selection state. The light-green area depicts an extremely small regime which was used to evaluate the spin-dependent deviation $\delta$. Green and red lines show different examples of sub-diffraction limited deflection. Inset emphasizes the linear regime of the measured gravity center of the post-selected beam $\delta_{m}(\Delta\theta)$. 
Fig. S7. Measured $\delta_M$ as a function of the QWP angle in post-selection state. The light-blue regime used to derive the actual spin-dependent deflection in units of degrees.
Fig. S8. Experimental amplification of the spin-dependent deflection. Measured momentum-space intensity distributions for GPMs with defect concentrates of $\xi = (a) \ 0.4$, (b) $0.5$, and (c) $0.7$. For each case, the measurements are taken at different experimental post-selection QWP angles $(\Delta \theta)$. Top and middle rows represent intensity measurements of $I(\Delta \theta)$ and $I(-\Delta \theta)$, respectively. Bottom row depicts the intensity subtraction $I(-\Delta \theta) - I(\Delta \theta)$. Black and white bars present the diffraction limit in momentum space.
Fig. S9. Random spin-split modes in the momentum space. (a) Calculated and (b) measured intensity difference $I_{\sigma_+}(\mathbf{k}) - I_{\sigma_-}(\mathbf{k})$ in momentum space. The defect concentration for the metasurface is $\zeta = 1$. Spin-up ($\sigma_+$) and spin-down ($\sigma_-$) modes are shown in red and blue, respectively. Black arrows highlight the selected examples of the spin-split modes. ((c) to (f)) Calculated ((c) and (e)) and measured ((d) and (f)) correlation functions of the momentum space intensities $I_{\sigma_+}(\mathbf{k})$ and $I_{\sigma_-}(\mathbf{k})$ ((c) and (d)), and for $I_{\sigma_+}(\mathbf{k})$ and $I_{\sigma_-}(\mathbf{k})$ ((e) and (f)). A single peak, the width of which is equal to the diffraction-limited spot, is clearly observed in cross-section ((c) and (d)).
Fig. S10. VP model. (a and b) Comparison of the spin-dependent phase distributions between the bound VP obtained from the calculated near field of a GPM (a) and the VP model (b).